پارمیس زندی

سؤال ۱

$8\cos x - \tan^2 x = 1 \qquad [0, 2\pi] \qquad \frac{8\cos^3 x - \sin^2 x}{\cos^2 x} = 1 \Rightarrow$

$\underbrace{\tan^2 x}_{\frac{\sin^2 x}{\cos^2 x}}$

$8\cos^3 x - \sin^2 x = \cos^2 x \Rightarrow 8\cos^3 x - (1-\cos^2 x) = \cos^2 x \Rightarrow$

$8\cos^3 x - 1 + \cancel{\cos^2 x} = \cancel{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ ۲ جواب

سؤال ۲

$5\sin^2(x) + 2\cos(3x) = -2 \qquad [-\pi, \pi]$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2 \Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2$

$\Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

اگر $x = \pi \Rightarrow \underbrace{-8-5}_{-13} + \underbrace{6+7}_{13} = 0$ ✓

اگر $x = -\pi \Rightarrow -8-5+6+7 = 0$ ✓

$\boxed{\text{۲ جواب}}$

سؤال ۳

$2\sin(x)\cos(2x) + \sin x = 1 \qquad [0, 2\pi]$

$\sin x(2\cos(2x)+1) = 1 \Rightarrow \sin x(\underbrace{1 - 2\sin^2 x + 1}_{2 - 2\sin^2 x}) = 1 \Rightarrow 2\sin x(\underbrace{1-\sin^2 x}_{\cos^2 x}) = 1$

$\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow 2\sin x\cos^2 x = 1$

سؤال ۴

$\frac{1}{\sin(\frac{\pi+4x}{2})} + \frac{1}{\cos(\frac{\pi+8x}{2})} = 0 \qquad [0, \pi] \qquad \tan 2\alpha = ?$

$\sin(\frac{\pi+4x}{2}) = \sin(\frac{\pi}{2}+2x) = \cos 2x$

$\cos(\frac{\pi+8x}{2}) = \cos(\frac{\pi}{2}+4x) = -\sin 4x$

$\frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \Rightarrow$

$\Rightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \Rightarrow \sin 4x = \cos 2x \Rightarrow 2\sin 2x\cos 2x = \cos 2x \Rightarrow$

$\cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0$ ✗ ق.ن

$2\sin 2x - 1 = 0 \Rightarrow \sin 2x = \frac{1}{2}$

$2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{12}$

$2x = 2k\pi + \frac{5\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{12}$

$[0, \pi]$: $x_1 = \frac{\pi}{12}$, $x_2 = \frac{5\pi}{12}$

$\Rightarrow x_2 - x_1 = \frac{5\pi}{12} - \frac{\pi}{12} = \boxed{\frac{\pi}{3} = \alpha} \qquad \boxed{\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}}$

سؤال ۵

$$m(\cos x - \sin x) - 2\sqrt{6}\sin 2x = \sqrt{6}$$

اگر $\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}}$ $\quad m = ?$

$$m\cos x - m\sin x - 2\sqrt{6}\sin 2x = \sqrt{6}$$

سؤال ۶

$$\sin(\underbrace{x+\frac{\pi}{6}}_{A})\cos(\underbrace{x-\frac{\pi}{3}}_{B}) = 1 , \quad [0, 2\pi]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$A = x+\frac{\pi}{6} \quad B = x-\frac{\pi}{3} \quad \begin{cases} A+B = x+\frac{\pi}{6}+x-\frac{\pi}{3} = 2x-\frac{\pi}{6} \\ A-B = (x+\frac{\pi}{6})-(x-\frac{\pi}{3}) = \frac{\pi}{2} \end{cases}$$

$$\Rightarrow \frac{1}{2}\left[\sin(2x-\frac{\pi}{6}) + \underset{1}{\cancel{\sin\frac{\pi}{2}}}\right] = 1 \Rightarrow \sin(2x-\frac{\pi}{6}) + 1 = 2 \Rightarrow$$

$$\sin(\underline{2x-\frac{\pi}{6}}) = 1 \Rightarrow 2x-\frac{\pi}{6} = 2k\pi+\frac{\pi}{2} \Rightarrow 2x = 2k\pi+\frac{\pi}{2}+\frac{\pi}{6}$$

$$\Rightarrow 2x = 2k\pi + \frac{3\pi+\pi}{6} \Rightarrow 2x = 2k\pi + \overset{\frac{4\pi}{6}}{\frac{2\pi}{3}} \Rightarrow x = k\pi+\frac{\pi}{3}$$

$$\xRightarrow{k=0,1} \boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$$

۲ جواب

سؤال ۷

$$\underset{a}{1}\sin x + \underset{b}{\sqrt{3}}\cos x = \underset{c}{\sqrt{2}} \qquad [-\pi, 2\pi]$$

$$\sqrt{1+3}\,\sin(x+\alpha) \Rightarrow 2\sin(x+\alpha) = \sqrt{2} \Rightarrow \sin(x+\alpha) = \frac{\sqrt{2}}{2}$$

$$\tan\alpha = \frac{b}{a} = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3} \Rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$$

$$x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \Rightarrow x = 2k\pi+\frac{\pi}{4}-\frac{\pi}{3} \Rightarrow x = 2k\pi-\frac{\pi}{12}$$

$$x+\frac{\pi}{3} = 2k\pi+\pi-\frac{\pi}{4} \Rightarrow x = 2k\pi+\pi-\frac{\pi}{4}-\frac{\pi}{3} \Rightarrow x = 2k\pi+\frac{5\pi}{12}$$

$x = -\frac{\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12} \Rightarrow$ جمع جواب‌ها $= -\frac{\pi}{12}+\frac{5\pi}{12}+\frac{23\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$

سوال ۸

$4\sin x \sin\left(\frac{3\pi}{2}-x\right)=1 \quad [0,2\pi]$

$$\underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x}$$

$$-4\sin x\cos x=1 \Rightarrow \sin x\cos x=-\frac{1}{4} \Rightarrow \frac{1}{2}\sin 2x=-\frac{1}{4} \Rightarrow \sin 2x=\frac{-\frac{1}{4}}{\frac{1}{2}}$$

$$\Rightarrow \sin 2x=-\frac{1}{2} \Rightarrow \begin{cases} 2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \\ 2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12} \end{cases}$$

(1) $k=0 \to x=-\frac{\pi}{12}$ ✗ $\quad k=1 \Rightarrow x=\pi-\frac{\pi}{12}=\frac{11\pi}{12}$ ✓ $\quad k=2 \Rightarrow x=\frac{23\pi}{12}$ ✓

(2) $k=0 \to x=\frac{7\pi}{12}$ ✓ $\quad k=1 \Rightarrow x=\frac{19\pi}{12}$ ✓

$\boxed{\text{مجموع} = 5\pi}$

سوال ۹

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \qquad A=$ مجموعه جواب $[0,\pi]$

$\max_A=?$

$$\left.\begin{aligned} 1+\cos 2\alpha&=2\cos^2\alpha \\ 1+\cos 4\alpha&=2\cos^2(2\alpha) \\ 1+\cos 8\alpha&=2\cos^2(4\alpha) \end{aligned}\right\} \begin{aligned} &2\cos^2\alpha\cdot 2\cos^2(2\alpha)\cdot 2\cos^2(4\alpha) \\ &=8(\cos^2\alpha\cdot\cos^2(2\alpha)\cdot\cos^2(4\alpha))=\frac{1}{8} \end{aligned}$$

$$\Rightarrow \cos^2\alpha\cdot\cos^2(2\alpha)\cdot\cos^2(4\alpha)=\frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cdot\cos(2\alpha)\cdot\cos(4\alpha)=\frac{1}{8}$$

$\alpha=\frac{\pi}{3}$

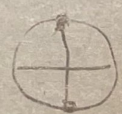
سوال ۱۰

$\tan(3x)\cdot\tan(x)=1 \quad [\pi,2\pi]$

$$\frac{\sin 3x}{\cos 3x}\cdot\frac{\sin x}{\cos x}=1 \Rightarrow \frac{\sin 3x\cdot\sin x}{\cos 3x\cdot\cos x}=1 \Rightarrow \sin 3x\sin x=\cos 3x\cos x$$

$$\Rightarrow \cos 3x\cos x-\sin 3x\cdot\sin x=0 \qquad \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\Rightarrow \cos(3x+x)=0 \Rightarrow \cos(4x)=0 \Rightarrow \begin{cases} 4x=k\pi\pm\frac{\pi}{2} \\ \Rightarrow x=\frac{k\pi}{4}\pm\frac{\pi}{8} \end{cases}$$



$$x=\frac{k\pi}{4}\pm\frac{\pi}{8} \to x=\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

مجموع جواب $=\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=\boxed{6\pi}$