۱۶,۵

پارمیس زندی

سؤال ۱

$8\cos x - \underbrace{\tan^2 x}_{\frac{\sin^2 x}{\cos^2 x}} = 1 \qquad [0, 2\pi] \qquad \frac{8\cos^3 x - \sin^2 x}{\cos^2 x} = 1 \Rightarrow$

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$8\cos^3 x - \sin^2 x = \cos^2 x \Rightarrow 8\cos^3 x - (1-\cos^2 x) = \cos^2 x \Rightarrow$

$8\cos^3 x - 1 + \cancel{\cos^2 x} = \cancel{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ ✓ ۲ جواب

سؤال ۲

$5\sin^2(x) + 2\cos(3x) = -2 \qquad [-\pi, \pi]$

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$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2 \Rightarrow 5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2$ ✓

$\Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

اگر $x = \pi \Rightarrow \underbrace{-8-5}_{-13} + \underbrace{6+7}_{13} = 0$ ✓

$\boxed{\text{۲ جواب}}$

اگر $x = -\pi \Rightarrow -8-5+6+7 = 0$ ✓

سؤال ۳

$2\sin(x)\cos(2x) + \sin x = 1 \qquad [0, 2\pi]$

$\sin x(2\cos(2x)+1) = 1 \Rightarrow \sin x(\underbrace{1-2\sin^2 x+1}_{2-2\sin^2 x}) = 1 \Rightarrow 2\sin x(\underbrace{1-\sin^2 x}_{\cos^2 x}) = 1$ ۱

$\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow 2\sin x\cos^2 x = 1$

سؤال ۴

$\frac{1}{\sin(\frac{\pi+4x}{2})} + \frac{1}{\cos(\frac{\pi+8x}{2})} = 0 \qquad [0, \pi] \qquad \tan 2\alpha = ?$

$\sin(\frac{\pi+4x}{2}) = \sin(\frac{\pi}{2}+2x) = \cos 2x$

$\cos(\frac{\pi+8x}{2}) = \cos(\frac{\pi}{2}+4x) = -\sin 4x$

$\frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0 \Rightarrow$

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$\Rightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \Rightarrow \sin 4x = \cos 2x \Rightarrow 2\sin 2x\cos 2x = \cos 2x \Rightarrow$

$\cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0$ ✗ ق.ن

$2\sin 2x - 1 = 0 \Rightarrow \sin 2x = \frac{1}{2}$

$2x = 2k\pi + \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{12}$

$2x = 2k\pi + \frac{5\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{12}$

$[0, \pi]$: $x_1 = \frac{\pi}{12}$, $x_2 = \frac{5\pi}{12}$

$\Rightarrow x_2 - x_1 = \frac{5\pi}{12} - \frac{\pi}{12} = \boxed{\frac{\pi}{3} = \alpha}$ ✓ $\boxed{\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}}$

سوال ۵ ۱۵

$m(\cos x - \sin x) - ۲\sqrt{۶}\,\sin ۲x = \sqrt{۶}$

اگر $\cos(x + \frac{\pi}{۴}) = \frac{۱}{\sqrt{۳}}$ $\quad m = ?$

$m\cos x - m\sin x - ۲\sqrt{۶}\,\sin ۲x = \sqrt{۶}$

سوال ۶

$\sin(\underbrace{x + \frac{\pi}{۶}}_{A})\cos(\underbrace{x - \frac{\pi}{۳}}_{B}) = ۱ \quad [۰, ۲\pi]$

$\sin A \cos B = \frac{۱}{۲}\left[\sin(A+B) + \sin(A-B)\right]$

$A = x + \frac{\pi}{۶} \quad B = x - \frac{\pi}{۳} \quad \begin{cases} A + B = x + \frac{\pi}{۶} + x - \frac{\pi}{۳} = ۲x - \frac{\pi}{۶} \\ A - B = (x + \frac{\pi}{۶}) - (x - \frac{\pi}{۳}) = \frac{\pi}{۲} \end{cases}$

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$\Rightarrow \frac{۱}{۲}\left[\sin(۲x - \frac{\pi}{۶}) + \cancel{\sin\frac{\pi}{۲}}^{۱}\right] = ۱ \Rightarrow \sin(۲x - \frac{\pi}{۶}) + ۱ = ۲ \Rightarrow$

$\sin(۲x - \frac{\pi}{۶}) = ۱ \Rightarrow ۲x - \frac{\pi}{۶} = ۲k\pi + \frac{\pi}{۲} \Rightarrow ۲x = ۲k\pi + \frac{\pi}{۲} + \frac{\pi}{۶}$

$\Rightarrow ۲x = ۲k\pi + \frac{۳\pi + \pi}{۶} \Rightarrow ۲x = ۲k\pi + \frac{۲\pi}{۳} \Rightarrow x = k\pi + \frac{\pi}{۳}$

$\xrightarrow{k = ۰, ۱} \boxed{x = \frac{\pi}{۳}, \frac{۴\pi}{۳}}$ ۲ جواب

سوال ۷

$\underbrace{۱}_{a}\sin x + \underbrace{\sqrt{۳}}_{b}\cos x = \underbrace{\sqrt{۲}}_{c} \quad [-\pi, ۲\pi]$

$\sqrt{۱ + ۳}\,\sin(x + \alpha) \Rightarrow ۲\sin(x+\alpha) = \sqrt{۲} \Rightarrow \sin(x+\alpha) = \frac{\sqrt{۲}}{۲}$

$\tan\alpha = \frac{b}{a} = \sqrt{۳} \Rightarrow \alpha = \frac{\pi}{۳} \Rightarrow \sin(x + \frac{\pi}{۳}) = \frac{\sqrt{۲}}{۲}$

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$x + \frac{\pi}{۳} = ۲k\pi + \frac{\pi}{۴} \Rightarrow x = ۲k\pi + \frac{\pi}{۴} - \frac{\pi}{۳} \Rightarrow x = ۲k\pi - \frac{\pi}{۱۲}$

$x + \frac{\pi}{۳} = ۲k\pi + \pi - \frac{\pi}{۴} \Rightarrow x = ۲k\pi + \pi - \frac{\pi}{۴} - \frac{\pi}{۳} \Rightarrow x = ۲k\pi + \frac{۵\pi}{۱۲}$

$x = -\frac{\pi}{۱۲}, \frac{۵\pi}{۱۲}, \frac{۲۳\pi}{۱۲} \Rightarrow$ جمع جواب‌ها $= -\frac{\pi}{۱۲} + \frac{۵\pi}{۱۲} + \frac{۲۳\pi}{۱۲} = \frac{۲۷\pi}{۱۲} = \boxed{\frac{۹\pi}{۴}}$

سوال ۸

$4\sin x \, \underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x}=1 \qquad [0,2\pi]$

$-4\sin x\cos x=1 \Rightarrow \sin x\cos x=-\frac{1}{4} \Rightarrow \frac{1}{2}\sin 2x=-\frac{1}{4} \Rightarrow \sin 2x=\frac{-\frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow \sin 2x=-\frac{1}{2} \Rightarrow \begin{cases} 2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \\ 2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12} \end{cases}$

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(1) $k=0 \rightarrow x=-\pi/12$ ✗ $\quad k=1 \Rightarrow x=\pi-\frac{\pi}{12}=\frac{11\pi}{12}$ ✓ $\quad k=2 \Rightarrow x=\frac{23\pi}{12}$ ✓

(2) $k=0 \rightarrow x=\frac{7\pi}{12}$ ✓ $\quad k=1 \Rightarrow x=\frac{19\pi}{12}$ ✓

$\boxed{\text{مجموع} = 5\pi}$ ✓

سوال ۹

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \qquad A=$ مجموع جواب $[0,\pi]$

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$\max_A = ?$

$\left.\begin{aligned} 1+\cos 2\alpha &= 2\cos^2\alpha \\ 1+\cos 4\alpha &= 2\cos^2(2\alpha) \\ 1+\cos 8\alpha &= 2\cos^2(4\alpha) \end{aligned}\right\} \begin{aligned} &2\cos^2\alpha \cdot 2\cos^2(2\alpha) \cdot 2\cos^2(4\alpha) \\ &= 8\left(\cos^2\alpha \cdot \cos^2(2\alpha)\cdot\cos^2(4\alpha)\right)=\frac{1}{8} \end{aligned}$

$\Rightarrow \cos^2\alpha \cdot \cos^2(2\alpha) \cdot \cos^2(4\alpha)=\frac{1}{64} \xrightarrow{\sqrt{}} \cos\alpha \cdot \cos(2\alpha)\cdot\cos(4\alpha)=\frac{1}{8}$

$\alpha=\frac{\pi}{3}$

سوال ۱۰

$\tan(3x)\cdot\tan(x)=1 \qquad [\pi,2\pi]$

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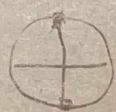
$\frac{\sin 3x}{\cos 3x}\cdot\frac{\sin x}{\cos x}=1 \Rightarrow \frac{\sin 3x \cdot \sin x}{\cos 3x \cdot \cos x}=1 \Rightarrow \sin 3x \sin x=\cos 3x \cos x$

$\Rightarrow \cos 3x \cos x-\sin 3x \cdot \sin x=0 \qquad \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\Rightarrow \cos(3x+x)=0 \Rightarrow \cos(4x)=0 \Rightarrow \begin{cases} 4x=k\pi\pm\frac{\pi}{2} \\ \Rightarrow x=\frac{k\pi}{4}\pm\frac{\pi}{8} \end{cases}$



$x=\frac{k\pi}{4}\pm\frac{\pi}{8} \longrightarrow x=\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

مجموع جواب‌ها $=\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=\boxed{6\pi}$

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$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k=0 \to x = \frac{\pi}{6}$, $k=1 \to x = \frac{5\pi}{6}$, $k=2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$

$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$

$$1+\cos 2x = 2\cos^2 x \;\bigstar$$

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \text{؟} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \;\bigstar}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \;\text{یا}\; 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \;\text{یا}\; 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

$\bigstar$ A نمی‌تواند برابر خود π باشد چون طبق $\bigstar$ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!