Date: Sub:

۱۷٫۵

باران تسایی مثلثات (۱۷)

۱- $8\cos x - \tan^2 x = 1$

$$\rightarrow \frac{8\cos^3 x - \sin^2 x}{\cos^2 x} = 1 \rightarrow 8\cos^3 x - \sin^2 x = \cos^2 x$$

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$$\rightarrow 8\cos^3 x = \underbrace{\sin^2 x + \cos^2 x}_{1} \rightarrow \cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2}$$

جواب ها در $[0, 2\pi] \leftarrow \frac{\pi}{3}, \frac{5\pi}{3}$ می‌شود ۲ جواب

۲- $5\underbrace{\sin^2 x}_{1-\cos^2 x} + 2\underbrace{\cos 3x}_{4\cos^3 x - 3\cos x} = -2$

$$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x = -2$$

$\cos x = t$

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$$8t^3 - 5t^2 - 6t + 7 = 0$$

$$(t+1)(\underbrace{8t^2 - 13t + 7}_{\Delta < 0}) \rightarrow t = -1$$

$$\cos x = -1$$

در $[-\pi, \pi]$ می‌شود $-\pi$ و π (۲ جواب)

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$$2\sin x \underbrace{\cos 2x}_{-2\sin^2 x+1} + \sin x = 1$$ -۳

$$-4\sin^3 x + 3\sin x - 1 = 0$$

$$\sin x = t \rightarrow -4t^3 + 3t - 1 = 0 \quad \left[\begin{array}{l} t = -1 \\ t = \frac{1}{2} \end{array}\right.$$ ۲

$$\sin x = -1$$

$$\sin x = \frac{1}{2}$$

جواب ها در $[0, 2\pi]$: $\frac{3\pi}{2}$ ، $\frac{\pi}{6}$ ، $\frac{5\pi}{6}$

مجموع: $\frac{5\pi}{6} + \frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{2}$

PITICO

-۴

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0$$

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$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)}=-\frac{1}{\cos\left(\frac{\pi+8x}{2}\right)}$$

$$-\cos\left(\frac{\pi+8x}{2}\right)=\sin\left(\frac{\pi+4x}{2}\right)$$

$$-\sin\left(\frac{\pi}{2}-\frac{\pi+8x}{2}\right)=\sin\left(\frac{\pi+4x}{2}\right)$$

$$-\sin\left(\frac{-8x}{2}\right)=\sin\left(\frac{\pi+4x}{2}\right)$$

$$\sin 4x=\sin\left(\frac{\pi}{2}+2x\right)$$

$$\to 4x=2k\pi+\frac{\pi}{2}+2x$$

$$2x=2k\pi+\frac{\pi}{2}\to x=k\pi+\frac{\pi}{4}$$

در بازه‌ی $[0,\pi]$ $\leftarrow$ $\frac{\pi}{4},\frac{3\pi}{4}$

اضافه $\to \frac{\pi}{2}=\alpha \to \tan 2\alpha=\tan\pi$ 0

$$m(\cos x-\sin x)-3\sqrt{6}\sin 2x=\sqrt{6}$$

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$$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

مقدار m؟

$$m\frac{\sqrt{6}}{3}-3\sqrt{6}\sin 2x=\sqrt{6}$$

$$\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x=\frac{1}{\sqrt{3}}$$

$$\sqrt{6}\left(\frac{m}{3}-3\sin 2x\right)=\cancel{\sqrt{6}}\,2$$

$$\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$$

$$\frac{m}{3}-1=1$$

$$\to \frac{m}{3}=2\to \boxed{m=6}$$

$$\cos x-\sin x=\frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{2\sqrt{6}}{6}=\frac{\sqrt{6}}{3}\to \cancel{\cos x=\frac{\sqrt{6}}{3}+\sin x}$$

$$\cancel{\cos^2x+\sin^2x=1}\qquad(\cos x-\sin x)^2=\frac{6}{9}=\frac{2}{3}$$

$$\cancel{\cos^2x}\qquad\sin^2x+\cos^2x-2\sin x\cos x$$

$$1-\sin 2x=\frac{2}{3}\to\sin 2x=\frac{1}{3}$$

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1$$

۶-

$$\left(\sin x\times\frac{\sqrt{3}}{2}+\cos x\times\frac{1}{2}\right)\left(\cos x\times\frac{1}{2}+\underbrace{\sin x\left(\frac{-\sqrt{3}}{2}\right)}\right)=1$$

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$$\frac{1}{4}\cos^2 x-\frac{3}{4}\sin^2 x=1-\frac{\sqrt{3}}{2}\sin x$$

$$-\begin{cases}\sin^2 x+\cos^2 x=1\\ -3\sin^2 x+\cos^2 x=4\end{cases}$$

$$4\sin^2 x=-3 \quad \sin^2 x=\frac{-3}{4}\longrightarrow$$

جواب ندارد

$\sin x + \sqrt{3}\cos x = \sqrt{2} \qquad [-\pi, 2\pi]?$ 7-

~~[illegible]~~ $\frac{|a|}{a}\sqrt{a^2+b^2}\,\sin(x+\alpha)$

~~[illegible]~~ $\hookrightarrow 2\sin(x+\frac{\pi}{3}) = \sqrt{2}$

$\tan\alpha = \sqrt{3} \longrightarrow \alpha = \frac{\pi}{3}$ 1,5

$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \rightsquigarrow x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4}$

$x = 2k\pi - \frac{\pi}{12}$

ج. $\rightarrow 2\pi - \frac{\pi}{12} \quad - \frac{\pi}{12}$ ، $\underset{k=0}{-\frac{\pi}{12}}$ ، $\underset{k=1}{2\pi - \frac{\pi}{12}}$ س

$\hookrightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ 5?

PITICO

دانتین موضوع -- تاریخ / /

۸ـ $4\sin x \sin(\underbrace{\frac{3\pi}{2} - x}) = 1$

$= \cos x$

$= 4\sin x \cos x$

$= 2\sin 2x = 1$

۲

$\sin 2x = \frac{-1}{2}$ $[0, 2\pi]$

$2x = \frac{7\pi}{6}$ و $\frac{11\pi}{6}$

$\hookrightarrow x = \frac{7\pi}{3}$ و $\frac{11\pi}{3}$

جمع $\longrightarrow \frac{7\pi}{3} + \frac{11\pi}{3} = \frac{18\pi}{3} = 6\pi$

۹- $(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$

$1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$ 1

$\Rightarrow \cos\frac{\alpha}{2}\cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8\sqrt{2}}$

$\alpha_{max} = \frac{\pi}{3}$

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$1+\cos 2x = 2\cos^2 x$

$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$

$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha\neq 0]{\times\sin\alpha}$ *

$\underbrace{(\sin\alpha\cos\alpha)}_{\frac{1}{2}\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$

$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$

$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$ $\rightarrow \boxed{\max A = \frac{8\pi}{9}}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفتیم!

دانتین موضوع $[\pi, 2\pi]$ تاریخ / /

$$\tan 3x \tan x = 1$$

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$$\frac{\sin 3x}{\cos 3x} \times \frac{\sin x}{\cos x}$$

$$\frac{\sin 3x \sin x}{\cos x \cos 3x} = 1$$

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$$\cos 3x \cos x - \sin 3x \sin x = 0$$

$$\cos 4x = 0 \qquad 4x = 2k\pi \pm \frac{\pi}{2}$$

$$x = \frac{k\pi}{2} \pm \frac{\pi}{8}$$

$$x \rightarrow \pi + \cancel{\frac{\pi}{8}},\ \underbrace{\frac{3\pi}{2} - \cancel{\frac{\pi}{8}},\ \frac{3\pi}{2} + \cancel{\frac{\pi}{8}}}_{3\pi},\ 2\pi - \cancel{\frac{\pi}{8}}$$

$\rightarrow$ 6π

۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0, 1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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$$\begin{cases} \sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x \end{cases} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \longrightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$$\cos 2x - 2\sin 2x \cos 2x = 0 \longrightarrow \cos 2x(1-2\sin 2x) = 0 \begin{cases} \cos 2x = 0 \ \times \\ \sin 2x = \frac{1}{2} \ \checkmark \end{cases}$$

★ اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \longrightarrow x = k\pi + \frac{\pi}{12} \text{ یا } k\pi + \frac{5\pi}{12}$$

$$\xrightarrow{0 \le x \le \pi} k=0 \longrightarrow x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12} \longrightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3} \longrightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$$

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$$\cos(x - \frac{\pi}{3}) = \cos(\frac{\pi}{3} - x)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos(\frac{\pi}{3} - x) = \sin(x + \frac{\pi}{6})$$

$$\sin^2(x+\frac{\pi}{6}) = 1 \longrightarrow \sin(x+\frac{\pi}{6}) = \pm 1 \longrightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3} \ , \ k=1 \ , \ x = \frac{4\pi}{3}$$

$\boxed{\text{معادله ۲ جواب در بازه دارد}}$