سارینا عابدینی

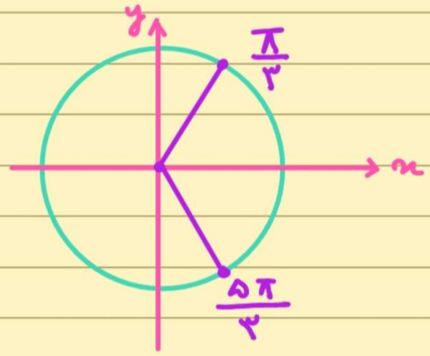
۱۹ آفرین

۱ـ

$8\cos x - \tan^2 x = 1$

$8\cos x = \cancel{\tan^2 x + 1}$ $\frac{1}{\cos^2 x}$ $\xrightarrow{\times \cos^2 x} 8\cos^3 x = 1 \to \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\cos x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{x\in[0,2\pi]} x = \frac{\pi}{3}\ ,\ x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



۲ـ

$5\sin^2 x + 2\cos 3x = -2$

نامنفی　　نامنفی

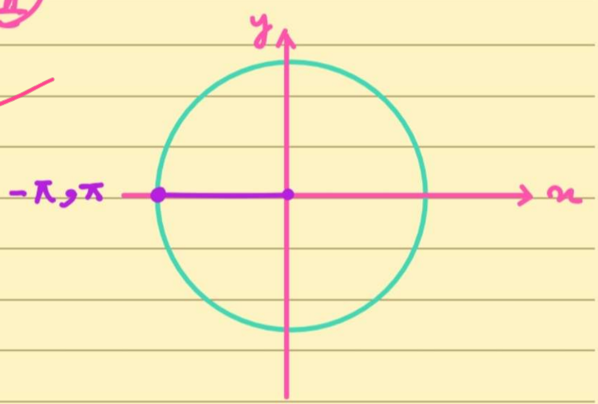
$5\sin^2 x + 2(\cos 3x + 1) = \cancel{5\sin^2 x} + \cancel{2\cos^2\frac{3x}{2}} = 0$ ← جمع دو عبارت نامنفی صفر شده پس هر یک عبارات برابر با صفر

$5\sin^2 x = 0 \longrightarrow \sin^2 x = 0 \longrightarrow \sin x = 0 \longrightarrow x \in \{-\pi, 0, \pi\}$ I

$2\cos^2\frac{3x}{2} = 0 \longrightarrow \cos\frac{3x}{2} = 0 \longrightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} = \frac{2k\pi}{3} + \frac{\pi}{3}$

$\longrightarrow x \in \{-\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi\}$ II

$I \cap II \longrightarrow x \in \{-\pi, \pi\}$



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۱٫۵

$$\overset{(1-2\sin^2 x)}{2\sin x\,\cancel{\cos(2x)}} + \sin x = 1$$

$$2\sin x - 4\sin^3 x + \sin x = 1 \longrightarrow \overset{\sin 3x}{\cancel{3\sin x - 4\sin^3 x}} = 1$$

$$\xrightarrow{x\in[0,2\pi]} 3x = \frac{\pi}{2} \longrightarrow x = \frac{\pi}{6}$$

۴-

$$\frac{1}{\underset{\underset{\cos 2x}{\cancel{\sin(\frac{\pi}{2}+2x)}}}{\cancel{\sin(\frac{\pi+4x}{2})}}} + \frac{1}{\underset{\underset{-\sin 4x}{\cancel{\cos(\frac{\pi}{2}+4x)}}}{\cancel{\cos(\frac{\pi+8x}{2})}}} = 0$$

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$$\longrightarrow \frac{\overset{-2\sin 2x\cos 2x}{\cancel{-\sin 4x}} + \cos 2x}{\cos 2x \sin 4x} = 0 \longrightarrow \cos 2x - 2\sin 2x \cos 2x = 0$$

$\cos 2x(1-2\sin 2x) = 0 \longrightarrow \cos 2x = 0 \longrightarrow$ مخرج صفر می‌شود.

$\qquad \longrightarrow 1 - 2\sin 2x = 0 \longrightarrow \sin 2x = \frac{1}{2}$

$$\xrightarrow{x\in[0,\pi]} 2x \in \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\} \longrightarrow x \in \left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\} \longrightarrow \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$$ ✓

۵-

$$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$$

$$\cos\left(x+\frac{\pi}{4}\right) = \cos x\cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$(\cos x - \sin x)^2 = \frac{2}{3} = 1 - \sin 2x = \frac{2}{3}$$ ✓

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ادامه‌ی ۵:

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$\sin 2x = \frac{1}{3}$

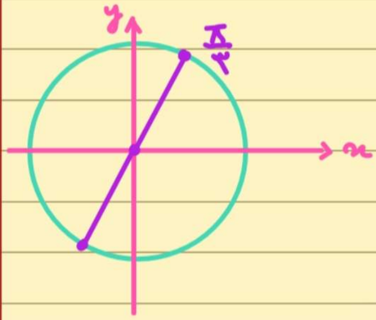
$\frac{\sqrt{2}}{\sqrt{3}} m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6} \longrightarrow \frac{\sqrt{2}}{\sqrt{3}} m = 2\sqrt{6} \longrightarrow m = 6$

۶-

$\sin(x+\frac{\pi}{6}) \underset{\sin(x+\frac{\pi}{6})}{\cancel{\cos(x-\frac{\pi}{3})}} = 1 \quad , \quad \frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2}$

$\sin^2(x+\frac{\pi}{6}) = 1 \longrightarrow \sin(x+\frac{\pi}{6}) = \pm 1 \xrightarrow{x\in[0,2\pi]} x + \frac{\pi}{6} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x \in \{\frac{\pi}{3}, \frac{4\pi}{3}\}$



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$\sin x + \sqrt{3}\cos x = \sqrt{2}$

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\frac{\pi}{6}\sin x + \cos\frac{\pi}{6}\cos x = \cos\frac{\pi}{4}$

$\cos(x-\frac{\pi}{6}) = \cos\frac{\pi}{4} \longrightarrow x - \frac{\pi}{6} = 2k\pi \pm \frac{\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12}$

$\hookrightarrow x = 2k\pi - \frac{\pi}{12}$

$\xrightarrow{x\in[-\pi, 2\pi]} -\frac{\pi}{4} + \frac{\pi}{2} + 2\pi = \frac{9\pi}{4}$

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۸-

۱٫۵

$4\sin x \,\cancel{\sin\left(\frac{3\pi}{2}-x\right)}^{-\cos x} = 1$

$-2\sin 2x = 1 \rightarrow \sin 2x = \frac{-1}{2} \xrightarrow{x\in[0,\pi]} 2x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$x \in \left\{\frac{7\pi}{12}, \frac{11\pi}{12}\right\}$

۹-

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8}$

$2\cos^2\alpha \cdot 2\cos^2 2\alpha \cdot 2\cos^2 4\alpha = \frac{1}{8} \rightarrow \cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64}$

$\cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\alpha\neq k\pi]{\times\sin\alpha} \underbrace{\underbrace{\underbrace{\cancel{\sin\alpha\cos\alpha}}_{\frac{1}{2}\sin 2\alpha}\cancel{\cos 2\alpha}}_{\frac{1}{4}\sin 4\alpha}\cancel{\cos 4\alpha}}_{\frac{1}{8}\sin 8\alpha} = \pm\frac{1}{8}(\sin\alpha)$

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$\frac{1}{8}\sin 8\alpha = \pm\frac{1}{8}\sin\alpha \longrightarrow \sin 8\alpha = \pm\sin\alpha$

$\sin 8\alpha = \sin\alpha \begin{cases} 8\alpha = 2k\pi + \alpha \Rightarrow \alpha = \frac{2k\pi}{7} \longrightarrow max = \frac{6\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \Rightarrow \alpha = \frac{2k\pi}{9} + \frac{\pi}{9} \rightarrow max = \frac{7\pi}{9} \end{cases}$

$\sin 8\alpha = -\sin\alpha = \sin(-\alpha) \begin{cases} 8\alpha = 2k\pi - \alpha \rightarrow \alpha = \frac{2k\pi}{9} \longrightarrow max = \frac{8\pi}{9} \\ 8\alpha = 2k\pi + \pi + \alpha \rightarrow \alpha = \frac{2k\pi}{7} + \frac{\pi}{7} \rightarrow max = \frac{5\pi}{7} \end{cases}$

$max = \frac{8\pi}{9}$

سارینا عابدینی

۱۵-

$\tan 3x \tan x = 1$

$1 - \tan 3x \tan x = 0 \longrightarrow \tan 4x = \dfrac{\tan 3x + \tan x}{1 - \tan 3x \tan x} = \pm\infty$ ۲

$4x = k\pi + \dfrac{\pi}{2} \longrightarrow x = \dfrac{k\pi}{4} + \dfrac{\pi}{8} \xrightarrow{x \in [\pi, 2\pi]} x \in \left\{\dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}\right\}$

$\dfrac{9\pi}{8} + \dfrac{11\pi}{8} + \dfrac{13\pi}{8} + \dfrac{15\pi}{8} = 6\pi$

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$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \longrightarrow \overbrace{3\sin x - 4\sin^3 x}^{\sin 3x} = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \dfrac{\pi}{2} \longrightarrow x = \dfrac{2k\pi}{3} + \dfrac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k = 0 \longrightarrow x = \dfrac{\pi}{6}$, $k = 1 \longrightarrow x = \dfrac{5\pi}{6}$, $k = 2 \longrightarrow x = \dfrac{9\pi}{6}$

$S = \dfrac{\pi}{6} + \dfrac{5\pi}{6} + \dfrac{9\pi}{6} = \dfrac{15\pi}{6} = \boxed{\dfrac{5\pi}{2}}$

۸

$$\sin\left(\frac{۳\pi}{۲}-x\right)=-\cos x$$

$$۴\sin x(-\cos x)=۱\longrightarrow -۲(۲\sin x\cos x)=۱\longrightarrow \sin ۲x=-\frac{۱}{۲}$$

$$۲x=۲k\pi-\frac{\pi}{۶}\ \text{یا}\ ۲k\pi+\frac{۷\pi}{۶}\longrightarrow x=\overset{2}{k\pi-\frac{\pi}{۱۲}}\ \text{یا}\ \overset{1}{k\pi+\frac{۷\pi}{۱۲}}$$

$$\xrightarrow{۰\leq x\leq ۲\pi}\begin{cases}\overset{1}{} & k=۰,۱\longrightarrow x=\frac{۷\pi}{۱۲},\frac{۱۹\pi}{۱۲}\\ \underset{2}{} & k=۱,۲\longrightarrow x=\frac{۱۱\pi}{۱۲},\frac{۲۳\pi}{۱۲}\end{cases}$$

$$S=\frac{۷\pi}{۱۲}+\frac{۱۹\pi}{۱۲}+\frac{۱۱\pi}{۱۲}+\frac{۲۳\pi}{۱۲}=\frac{۶۰\pi}{۱۲}=\boxed{۵\pi}$$