$-12+9$

$\frac{-4\times 5}{4}$

Subject: حدیث متین پور   تکلیف ۱۷   page : (   )

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۴ خوب تکلیف بنویس!

۱) $8\cos x - \tan^2 x = 1$

$8\cos x = 1 + \frac{1}{\cos^2 x} \longrightarrow 8\cos^3 x = 1 \quad \cos^3 x = \frac{1}{8} \quad \cos x = \frac{1}{2} \quad x = \frac{\pi}{3}$

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$x = 2k\pi \pm \frac{\pi}{3} \longrightarrow x = \frac{\pi}{3}, \quad x = \frac{5\pi}{3}$

۲) ۰

$1 - 2\sin^2 x$

۳) $2\sin x \cdot \cos 2x + \sin x = 1 \Rightarrow -4\sin^3 x + 3\sin x = 1$

$\sin(3x) = 1 \quad 3x = 2k\pi + \frac{\pi}{2}$

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$x = \frac{2k\pi}{3} + \frac{\pi}{6}$

$x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}, \quad x = \frac{3\pi}{2}$

$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6}$

۴) ۰

$\frac{1}{\sin(\frac{\pi}{?} + 2x)} + \frac{1}{\cos(\frac{\pi}{?} + 4x)} = 0$

- ۲

$\cos 3x = 2\cos^2\frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2(2\cos^2\frac{3x}{2} - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \le x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک دو مجموعه جواب $\{-\pi, \pi\}$ است

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$$\begin{cases} \sin(2x + \frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2} + 4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$\cos 2x - 2\sin 2x \cos 2x = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$

$\rightarrow \cos 2x = 0$ ✗

$\rightarrow \sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

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$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

۶

$$\cos\left(x-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right) = \sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1 \longrightarrow x+\frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3}, \quad k=1, \ x = \frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12}$$

$$x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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$$\sin\left(\frac{3\pi}{2}-x\right)=-\cos x$$

$$4\sin x(-\cos x)=1 \longrightarrow -2(2\sin x\cos x)=1 \longrightarrow \sin 2x=-\frac{1}{2}$$

$$2x=2k\pi-\frac{\pi}{6} \text{ یا } 2k\pi+\frac{7\pi}{6} \longrightarrow x=\overset{2}{k\pi-\frac{\pi}{12}} \text{ یا } \overset{1}{k\pi+\frac{7\pi}{12}}$$

$$\xrightarrow{0\leq x\leq 2\pi} \begin{cases} 1:\ k=0,1 \longrightarrow x=\frac{7\pi}{12},\ \frac{19\pi}{12} \\ 2:\ k=1,2 \longrightarrow x=\frac{11\pi}{12},\ \frac{23\pi}{12} \end{cases}$$

$$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

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$$\tan 3x=\frac{1}{\tan x}=\cot x \longrightarrow \tan 3x=\tan\left(\frac{\pi}{2}-x\right)$$

$$3x=k\pi+\frac{\pi}{2}-x \longrightarrow 4x=k\pi+\frac{\pi}{2} \longrightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S=\left(\frac{9+11+13+15}{8}\right)\pi=\frac{48\pi}{8}=\boxed{6\pi}$$

$$1+\cos 2x = 2\cos^2 x \quad \star$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ \star}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!