عل‌رضا

۱۸

Subject.

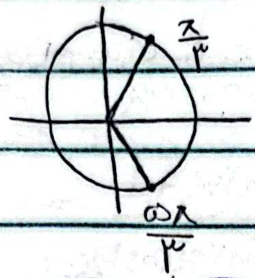
Date. / /

۱)

$8\cos x - \tan^2 x = 1 \to$

$8\cos x = 1 + \tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \to$

$\cos x = \frac{1}{2}$



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۲تا جواب

۲)

$5\sin^2 x + 2\cos 3x = -2 \Rightarrow 5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) + 2 = 0$

$5 - 5\cos^2 x + 8\cos^3 x - 6\cos x + 2 = 0 \Rightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\Rightarrow (\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\text{ریشه ندارد}} = 0 \Rightarrow \cos x = -1,$

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$x = -\pi, \pi$

۳)

$2\sin x \cos 2x + \sin x = 1 \to \sin x(2\cos 2x + 1) = 1$

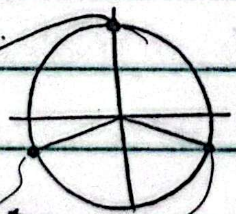
$\sin x(2(1 - 2\sin^2 x) + 1) = 1 \Rightarrow \sin x(3 - 4\sin^2 x) = 1 \Rightarrow$

۱/۵

$3\sin x - 4\sin^3 x = 1 \to 4\sin^3 x - 3\sin x - 1 = 0 \to$

$(\sin x - 1)(4\sin^2 x + 4\sin x + 1) = 0 \Rightarrow (\sin x - 1)(2\sin x + 1) = 0$

$\sin x = 1$ $\qquad$ $\sin x = 1$ $\qquad$ $\sin x = \frac{-1}{2}$



$\sin x = -\frac{1}{2}$

۳تا جواب

$\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

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(4)

$$\frac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \frac{1}{\cancel{8}\cos\left(\frac{\pi+8x}{2}\right)} = 0$$

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} =$$

2

$$\frac{1}{\cos 2x} + \frac{1}{-2\sin 2x\cos 2x} = \frac{-2\sin 2x+1}{-2\sin 2x\cos 2x} = 0 \qquad \sin 2x \neq 0 \quad \cos 2x \neq 0$$

$$-2\sin 2x+1=0 \longrightarrow +2\sin 2x = 1 \Rightarrow \sin 2x = 1/2 \Rightarrow \frac{\pi}{12}, \frac{5\pi}{12}$$

$$\frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \Rightarrow \tan 2 \times \frac{\pi}{3} = \tan \frac{\cancel{2\pi}}{3} \Rightarrow -\sqrt{3}$$

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(5) 1/5

$$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$$

$$\cos\left(\frac{\pi}{4}+x\right) = \cos x\frac{\sqrt{2}}{2} - \sin x\frac{\sqrt{2}}{2} \Rightarrow \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$$

$$\cos x - \sin x = \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{2}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

$$m\left(\frac{\sqrt{6}}{3}\right) - 3\sqrt{6}\sin 2x = \sqrt{6}$$

$$(\cos - \sin x)^2 = \cos^2 x + \sin^2 x - 2\cos x\sin x = 1 - 2\cos\sin x = \frac{1}{3} \Rightarrow$$

$$\frac{2}{3} = 2\sin x\cos x = \sin 2x$$

$$m\left(\frac{\sqrt{6}}{3} - 3\sqrt{6} \times \frac{2}{3} = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{3}m - 2\sqrt{6} = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{3}m = 3\sqrt{6}\right.$$

$m=9$

(۶)

$\sin(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})=1 \Rightarrow \sin(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=1$

$\sin^2(x+\frac{\pi}{4})=1 \longrightarrow \sin(x+\frac{\pi}{4})=\pm 1 \Rightarrow$ ۲ حالت داریم

$\sin(x+\frac{\pi}{4})=1 \longrightarrow x+\frac{\pi}{4}=\frac{\pi}{2} \Rightarrow x=\frac{4\pi}{12}=\frac{\pi}{3}$ ۲

$\sin(x+\frac{\pi}{4})=-1 \Rightarrow x+\frac{\pi}{4}=\frac{3\pi}{2} \Rightarrow x=\frac{8\pi}{6}=\frac{4\pi}{3}$

(۷)

$\sin x+\sqrt{3}\cos x=\sqrt{2} \Rightarrow \sin x=\sqrt{2}-\sqrt{3}\cos x \Rightarrow$

$\sin^2 x=2+3\cos^2 x-2\sqrt{6}\cos x \Rightarrow 1-\cos^2 x=2+3\cos^2 x-2\sqrt{6}\cos x$

$\Rightarrow 4\cos^2 x-2\sqrt{6}\cos x+1=0 \Rightarrow \frac{2\sqrt{6}\pm\sqrt{24-16}}{8}=$ ۲

$\frac{2\sqrt{6}\pm 2\sqrt{2}}{8} \begin{cases} \frac{\sqrt{6}-\sqrt{2}}{4}=75^\circ \\ \frac{\sqrt{6}+\sqrt{2}}{4}=-15, 345 \end{cases} \rightarrow 405^\circ$

(۸)

$4\sin x\sin(\frac{3\pi}{2}-x)=1 \Rightarrow -4\sin\alpha\cos\alpha=1 \Rightarrow$ ۱,۵

$-2\sin 2\alpha=1 \Rightarrow \sin 2\alpha=\frac{-1}{2} \Rightarrow \alpha=\frac{7\pi}{12}, \frac{11\pi}{12}$

(۹)

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \Rightarrow$

$(1+\cos 2\alpha)(1+2\cos^2 2\alpha-1)(1+2\cos^2 4\alpha-1)=\frac{1}{8} \Rightarrow$

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8} \Rightarrow \cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64}$

$\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8} \quad \times\sin$

$\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{\sin\alpha}{8} \qquad \sin 8\alpha=\sin\alpha$

$\frac{\sin 2\alpha}{2} \quad \frac{\sin 4\alpha}{4} \quad \frac{\sin 8\alpha}{8}=\pm\frac{1}{8} \qquad \sin 8\alpha=-\sin\alpha$

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$\max\alpha = \frac{8\pi}{9}$

$8\alpha = 2k\pi + \alpha \Rightarrow \alpha = \frac{2k\pi}{7}$

$8\alpha = 2k\pi + \pi - \alpha \Rightarrow \frac{2k\pi}{9} + \frac{\pi}{9}$

$\begin{cases} 8\alpha = 2k\pi - \alpha \Rightarrow \alpha = \frac{2k\pi}{9} \\ 8\alpha = 2k\pi + \pi + \alpha \Rightarrow \alpha = \frac{2k\pi}{7} + \frac{\pi}{7} \end{cases}$

$\tan 3x \times \tan x = 1 \Rightarrow \frac{\sin 3x \times \sin x}{\cos 3x \cos x} = 1 \Rightarrow$ (10)

(1,5)

$\sin 3x \sin x = \cos 3x \cos x \Rightarrow \cos 3x \cos x - \sin 3x \sin x \Rightarrow$

$\cos(3x + x) = 0 \Rightarrow \cos 4x = 0 \Rightarrow x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

S?

$\tan 3x = \frac{1}{\tan x} = \cot x \rightarrow \tan 3x = \tan(\frac{\pi}{2} - x)$ (10)

$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$S = (\frac{9+11+13+15}{8})\pi = \frac{48\pi}{8} = \boxed{4\pi}$

۳

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$

$k=0 \to x = \frac{\pi}{6}$, $k=1 \to x = \frac{5\pi}{6}$, $k=2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

۵

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$

$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$

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$\sin(\frac{3\pi}{2} - x) = -\cos x$

$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6}$ یا $2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}}$ یا $\overset{1}{k\pi + \frac{7\pi}{12}}$

$\xrightarrow{0 \leq x \leq 2\pi} \begin{cases} 1:\ k = 0, 1 \to x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ 2:\ k = 1, 2 \to x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$

$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$