کیمیا امیری نیا

۱۹ آزمون

①

$8\cos x - \tan^2 x = 1 \to 8\cos x = 1+\tan^2 x \to 8\cos x = \frac{1}{\cos^2 x} \to 8\cos^3 x = 1$

$\to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$

۲ جواب ✓

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②

$5\sin^2 x + 2\cos 3x = -2 \to 5\sin^2 x + 2\cos 3x + 2 = 0 \to 5\sin^2 x + 2(\cos 3x + 1) = 0$

$2\cos^2\frac{3x}{2}$ = فرمول طلایی

$5\sin^2 x + 2\cos^2\frac{3x}{2} = 0$ هر دو نامنفی هستند.

$\to \sin x = 0 \to x = k\pi$ ①

$\to \cos\frac{3x}{2} = 0 \to \frac{3x}{2} = k\pi + \frac{\pi}{2} \to 3x = 2k\pi + \pi \to x = \frac{2k\pi}{3} + \frac{\pi}{3}$ ②

$① \cap ② = \{\pi, -\pi\}$ ۲ جواب ✓

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③

$2\sin x \underbrace{\cos 2x}_{(1-2\sin^2 x)} + \sin x = 1 \to 4\sin^3 x - 3\sin x + 1 = 0 \to (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0$

$\sin x = -1$, $\sin x = \frac{1}{2}$

$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \to S = \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$

۱/۵

④

$\frac{1}{\sin(\frac{\pi+4x}{2})} + \frac{1}{\cos(\frac{\pi+8x}{2})} = 0 \to \frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0$, $\cos(\frac{\pi}{2}+4x) = -\sin(4x)$

$\frac{-\sin 4x + \sin(\frac{\pi}{2}+2x)}{-\sin 4x \cdot \sin(\frac{\pi}{2}+2x)} = 0 \to \sin(\frac{\pi}{2}+2x) = \sin 4x$

$4x = 2k\pi + \frac{\pi}{2} + 2x \to x = k\pi + \frac{\pi}{4}$

$4x = 2k\pi + \pi - \frac{\pi}{2} - 2x \to x = \frac{k\pi}{3} + \frac{\pi}{12}$

$x = \cancel{\frac{\pi}{4}}, \frac{\pi}{12}, \frac{5\pi}{12}, \cancel{\frac{9\pi}{12}}$ (جزو دامنه نیست) (جزو دامنه نیست)

ارفاق

$\tan\frac{2\pi}{3} = -\sqrt{3}$

$\alpha = \frac{4\pi}{12} = \frac{\pi}{3} \to 2\alpha = \frac{2\pi}{3} \to \tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

سوال فقط $\tan 2\alpha$ را می‌خواهد

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⑤

$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}} \to \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{1}{\sqrt{3}} \to \cos - \sin = \frac{2}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{3}}$

$(\cos - \sin)^2 = 1 - \sin 2x = (\frac{\sqrt{2}}{\sqrt{3}})^2 = \frac{2}{3} \to \sin 2x = \frac{1}{3} \to m(\frac{\sqrt{2}}{\sqrt{3}}) - \sqrt{6} = \sqrt{6} \Rightarrow m = 6$

$m\frac{\sqrt{2}}{\sqrt{3}} = 2\sqrt{6}$

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⑤

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1 \rightarrow 2\sin(x+\frac{\pi}{6})=1 \rightarrow \sin(x+\frac{\pi}{6})=\frac{1}{2}=\sin\frac{\pi}{6}$

$=\cos(\frac{\pi}{3}-x)$

$x+\frac{\pi}{6}=2k\pi+\frac{\pi}{6} \rightarrow x=2k\pi$

$x+\frac{\pi}{6}=2k\pi+\pi-\frac{\pi}{6} \rightarrow x=2k\pi+\frac{2\pi}{3}$

در $[0,2\pi)$

$x=0, 2\pi, \frac{2\pi}{3}$ ✓

✗ ✗

⑥

$\sin x+\sqrt{3}\cos x=\sqrt{2} \rightarrow \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2} \rightarrow \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}=\sin\frac{\pi}{4}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \rightarrow x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \rightarrow x=2k\pi+\frac{5\pi}{12}$

$x=-\frac{\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{12}$

$\frac{9\pi}{4}=\frac{27\pi}{12}=$ مجموع ✓

⑦

$4\sin x\cdot\sin(\frac{3\pi}{2}-x)=-4\sin x\cos x=1 \rightarrow -2\sin 2x=1 \rightarrow \sin 2x=-\frac{1}{2}=\sin(-\frac{\pi}{6})$

$2x=2k\pi-\frac{\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi+\frac{7\pi}{12}$

$x=\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}$

$9\pi=$ مجموع ✓

⑧

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8} \rightarrow (\cos^2\alpha\cdot\cos^2 2\alpha\cdot\cos^2 4\alpha)=\frac{1}{64}$ $\xrightarrow{\times \sin^2\alpha}$

$\frac{\frac{1}{64}\sin^2 8\alpha}{\sin^2\alpha}=\frac{1}{64} \rightarrow \sin^2 8\alpha=\sin^2\alpha \rightarrow \cos 16\alpha=\cos 2\alpha$

$16\alpha=2k\pi\pm 2\alpha \rightarrow \alpha=\frac{k\pi}{9}\pm\frac{\pi}{8}$

$\alpha=\frac{k\pi}{9}$

$\alpha=-\frac{k\pi}{7}$ $\rightarrow$ Max $=\frac{8\pi}{9}$ ✓

⑨

$\tan 3x\cdot\tan x=1$

$\tan 3x=\frac{1}{\tan x}=\cot x \rightarrow \tan 3x=\tan(\frac{\pi}{2}-x)$

$3x=k\pi+\frac{\pi}{2}-x \rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$

$x=\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$ $\longrightarrow$ مجموع $=\frac{48\pi}{8}=6\pi$ ✓

(۴)

$\cos(x-\frac{\pi}{2}) = \cos(\frac{\pi}{2}-x)$

$\frac{\pi}{2}-x+x+\frac{\pi}{4} = \frac{3\pi}{4}$ ⟶ $\cos(\frac{\pi}{2}-x) = \sin(x+\frac{\pi}{4})$

$\sin^2(x+\frac{\pi}{4}) = 1 \rightarrow \sin(x+\frac{\pi}{4}) = \pm 1 \rightarrow x+\frac{\pi}{4} = k\pi+\frac{\pi}{2}$

$x = k\pi+\frac{\pi}{4} \xrightarrow{0 \le x < 2\pi}$ $k=0 \rightarrow x=\frac{\pi}{4}$ ، $k=1, x=\frac{5\pi}{4}$

معادله ۲ جواب در بازه دارد

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$\cos 2x = 1-2\sin^2 x$ (فرمول طلایی)

$2\sin x(1-2\sin^2 x)+\sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \rightarrow 3x = 2k\pi+\frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3}+\frac{\pi}{6}$ $\xrightarrow{0 \le x < 2\pi}$

$k=0 \rightarrow x=\frac{\pi}{6}$ ، $k=1 \rightarrow x=\frac{5\pi}{6}$ ، $k=2 \rightarrow x=\frac{9\pi}{6}$

$S = \frac{\pi}{6}+\frac{5\pi}{6}+\frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$
