

$$\begin{aligned} 1 \cos x - \tan^2 x &= 1 \rightarrow 1 \cos x = 1 + \frac{\sin^2 x}{\cos^2 x} \rightarrow 1 \cos x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &\rightarrow 1 \cos x = \frac{1}{\cos^2 x} \rightarrow \cos^2 x = \frac{1}{1} \rightarrow \cos x = \pm \frac{1}{1} \xrightarrow{0 \leq x < 2\pi} x = \frac{\pi}{3}, \frac{5\pi}{3} \rightarrow \text{عدد ۱} \end{aligned}$$

$$\begin{aligned} 4 \sin^2 x + 2 \cos^2 x &= -2 \rightarrow 4(1 - \cos^2 x) + 2 \cos^2 x = -2 \\ 4 \cos^2 x - 4 \cos^2 x - 2 \cos^2 x + 4 &= -2 \rightarrow 2 \cos^2 x = 6 \rightarrow \cos^2 x = 3 \rightarrow \cos x = \pm \sqrt{3} \end{aligned}$$

جمع ضرایب بتواند
فرد با جمع ضرایب با
توان زوج با هم برابر است.

$$\begin{aligned} \cos x &= t \rightarrow 4t^2 - 4t^2 - 2t^2 + 4 = -2 \\ 4t^2 - 4t^2 - 2t^2 + 4 &= -2 \\ -2t^2 + 4 &= -2 \\ -2t^2 &= -6 \\ t^2 &= 3 \\ t &= \pm \sqrt{3} \end{aligned}$$

این عبارت را می توان
در مجموع، می توان
جواب داشت!

$$\begin{aligned} 2 \sin(x) \cos(x) + \sin(x) &= 1 \rightarrow 2 \sin(x) (1 - \sin^2 x) + \sin(x) = 1 \rightarrow 2 \sin(x) - 2 \sin^3 x + \sin(x) = 1 \\ 3 \sin(x) - 2 \sin^3 x &= 1 \rightarrow \sin^3 x = 1 \rightarrow x = \frac{\pi}{2} + 2k\pi \rightarrow x = \frac{\pi}{2} + \frac{2k\pi}{1} \\ 0 \leq x < 2\pi &\rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2} \end{aligned}$$

مجموع: $\frac{\pi}{2} + \frac{5\pi}{2} + \frac{9\pi}{2} = \frac{15\pi}{2}$

$$\begin{aligned} \frac{1}{\sin(\frac{\pi}{2} + x)} + \frac{1}{\cos(\frac{\pi}{2} + x)} &= \frac{1}{\cos x} - \frac{1}{\sin x} \rightarrow \cos x \sin x \\ \rightarrow \sin(\frac{\pi}{2} + x) &= \sin x \rightarrow x = \frac{\pi}{2} + 2k\pi \rightarrow x = \frac{\pi}{2} + \frac{2k\pi}{1} \\ \rightarrow x &= \frac{\pi}{2} + 2k\pi \rightarrow x = \frac{\pi}{2} + \frac{2k\pi}{1} \end{aligned}$$

چون هیچ راهی نداریم
غیر قابل قبول
قبل قبول

$$\tan \frac{\pi}{4} = -\sqrt{3}$$

$$\begin{aligned} \cos(x + \frac{\pi}{2}) &= \frac{1}{\sqrt{2}} \rightarrow \cos^2(x + \frac{\pi}{2}) = \frac{1 + \cos(2x + \pi)}{2} \rightarrow \frac{1}{2} = \frac{1 - \sin 2x}{2} \rightarrow \sin 2x = \frac{1}{2} \\ m(\cos x - \sin x) &= 2\sqrt{2} \sin 2x \sqrt{2} \rightarrow m x - \sqrt{2} \sin(x - \frac{\pi}{2}) - 2\sqrt{2} \times \frac{1}{2} = \sqrt{2} \\ \rightarrow m x &= \sqrt{2} \sin(x - \frac{\pi}{2}) + \sqrt{2} \rightarrow m x \cos(x + \frac{\pi}{2}) \times \sqrt{2} = \sqrt{2} \rightarrow m x \sqrt{2} \times \frac{1}{\sqrt{2}} = \sqrt{2} \\ m \sqrt{2} &= \sqrt{2} \rightarrow m = 1 \end{aligned}$$

$$\sin(x + \frac{\pi}{4}) \cos(x - \frac{\pi}{4}) = 1 \rightarrow \sin(x + \frac{\pi}{4}) \times \cos(\frac{\pi}{4} - x) = 1 \xrightarrow{\text{زنجیره مستقیم}} \sin(x + \frac{\pi}{4}) \sin(x + \frac{\pi}{4}) = 1$$

$$\sin(x + \frac{\pi}{4}) = \pm 1 \rightarrow x + \frac{\pi}{4} = k\pi \pm \frac{\pi}{4} \xrightarrow{\text{اگر } k \text{ فرد}} x = \frac{\pi}{4} \text{ و } \frac{5\pi}{4} \rightarrow \text{اگر } k \text{ زوج} \rightarrow x = \frac{3\pi}{4} \text{ و } \frac{7\pi}{4}$$

آنگاه ۴ جوابی داریم یا ۱۱ اهم باشد پس جوابها $\rightarrow$ به دست می آید و معادله ۲ جواب دارد!

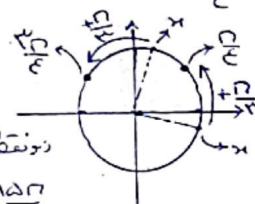
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$$\sin x + \sqrt{r} \cos x = \sqrt{r} \rightarrow \frac{a}{|a|} \sqrt{a^2 + b^2} \sin(x + \alpha) \text{ و } \tan \alpha = \frac{b}{a}$$

$$\tan \alpha = \sqrt{r} \rightarrow \alpha = \frac{\pi}{4} \rightarrow r \sin(x + \frac{\pi}{4}) = \sqrt{r} \rightarrow \sin(x + \frac{\pi}{4}) = \frac{\sqrt{r}}{r} = \sin \frac{\pi}{4} \pm \sin \frac{3\pi}{4}$$

$$\sin(-\pi/2) \rightarrow \text{طبق شکل، این نقطه است که از } \frac{\pi}{4} \text{ و } \frac{3\pi}{4} \text{ به فاصله } \frac{\pi}{2} \text{ به سمت } \frac{\pi}{2} \text{ می آید}$$

$$\frac{\pi}{4} + \frac{12\pi}{13} = \frac{13\pi}{13} = \pi \text{ (موجب نخواهد شد!)} \rightarrow \frac{\pi}{4} - \frac{12\pi}{13} = \frac{13\pi}{13} - \frac{12\pi}{13} = \frac{\pi}{13}$$



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$$r \sin x \sin(\frac{\pi}{4} - x) = 1 \rightarrow r \sin x \cos x = 1 \rightarrow -r \sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{r}$$

$$\sin 2x = \sin \frac{\pi}{4} \rightarrow 2x = k\pi + \frac{\pi}{4} \rightarrow x = k\pi + \frac{\pi}{8} \text{ و } x = \frac{\pi}{8} \text{ و } \pi + \frac{\pi}{8}$$

$$\rightarrow 2x = k\pi + \pi - \frac{\pi}{4} \rightarrow x = k\pi + \frac{3\pi}{8} \rightarrow x = \frac{11\pi}{8} \text{ و } \frac{15\pi}{8}$$

$$\text{مجموع: } \frac{\pi}{8} + \frac{11\pi}{8} + \frac{15\pi}{8} + \frac{3\pi}{8} = \frac{40\pi}{8} = 5\pi$$

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$$(1 + \cos 2\alpha)(1 + \cos 4\alpha)(1 + \cos 8\alpha) = \frac{1}{\lambda} \rightarrow (r \cos^2 \alpha)(r \cos^2 2\alpha)(r \cos^2 4\alpha) = \frac{1}{\lambda} \xrightarrow{\times \frac{\sin^2 \alpha}{\sin^2 \alpha}}$$

$$\frac{(\sin^2 \alpha)(r \cos^2 \alpha)(r \cos^2 2\alpha)(r \cos^2 4\alpha)}{\sin^2 \alpha} = \frac{1}{\lambda} \rightarrow \frac{(\sin^2 2\alpha)(r \cos^2 \alpha)(r \cos^2 4\alpha)}{r \sin^2 \alpha} = \frac{1}{\lambda} \rightarrow \frac{(\sin^2 4\alpha)(r \cos^2 2\alpha)}{r \sin^2 2\alpha}$$

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$$\rightarrow \frac{\sin^2 4\alpha}{\lambda \sin^2 2\alpha} = \frac{1}{\lambda} \rightarrow \sin^2 4\alpha = \sin^2 2\alpha \rightarrow \sin 4\alpha = \sin 2\alpha \rightarrow 4\alpha = k\pi + 2\alpha \rightarrow \alpha = \frac{k\pi}{2} \text{ و } 4\alpha = k\pi + \pi - 2\alpha \rightarrow \alpha = \frac{k\pi}{3} + \frac{\pi}{3}$$

$$\rightarrow \sin 4\alpha = -\sin 2\alpha \rightarrow \sin 4\alpha = \sin(-2\alpha) \rightarrow 4\alpha = k\pi - 2\alpha \rightarrow \alpha = \frac{k\pi}{3}$$

$$\rightarrow 4\alpha = k\pi + \pi + 2\alpha \rightarrow \alpha = \frac{k\pi}{3} + \frac{\pi}{2} \text{ و } 4\alpha = k\pi - \pi - 2\alpha \rightarrow \alpha = \frac{k\pi}{3} - \frac{\pi}{2}$$

$$\tan(2x) \times \tan x = 1 \rightarrow \tan 2x = \frac{1}{\tan x} \rightarrow \tan 2x = \cot x \rightarrow \tan 2x = \tan(\frac{\pi}{2} - x)$$

$$2x = k\pi + \frac{\pi}{2} - x \rightarrow 3x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{3} + \frac{\pi}{6} \xrightarrow{\text{اگر } k \text{ زوج}} x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\text{مجموع: } \frac{\pi}{6} + \frac{11\pi}{6} + \frac{7\pi}{6} + \frac{5\pi}{6} = \frac{24\pi}{6} = 4\pi$$

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