همسا_ بوطالعی

۱_

$8\cos x = \dfrac{\cancel{\tan^2 x+1}}{\dfrac{1}{\cos^2\alpha}} \Rightarrow$

$\dfrac{8\cos^3 x - 1}{\cos^2 x} = 0 \qquad 8\cos^3 x - 1 = 0 \qquad 8\cos^3 x = 1$

$\cos x = \dfrac{1}{2}$

2

$\cos^2 x \neq 0 \quad R - \{k\pi + \frac{\pi}{2}\} \qquad \boxed{2k\pi \pm \frac{\pi}{3}}$

۲_

$5(1-\cos^2 x) + 2\cos 3x = -2$

$5 - 5\cos^2 x + \cancel{2\cos 3x} = -2$

$2(4\cos^3\alpha - 3\cos\alpha)$

$5 - 5\cos^2\alpha + 8\cos^3\alpha - 6\cos\alpha = -2$

$8\cos^3\alpha - 5\cos^2\alpha - 6\cos\alpha + 7 = 0$

if $\Rightarrow \cos = -1$ در عبارت $= 0 \Rightarrow$ بخش پذیر $(\cos x+1)$ دارد

4

$8\cos^3 x + 8\cos^2 x - 13\cos x - 13\cos x + 7\cos x + 7$

$8\cos^2 x(\cos x+1) - 13\cos x(\cos x+1) + 7(\cos x+1)$

$(\cos x+1)(8\cos^2 x - 13\cos x + 7)$

$\Delta = 169 - 224 = -55 \qquad \Delta < 0 \Rightarrow$ فقط $\cos x + 1$ جواب می‌دهد

$\cos x = -1$

$k\pi + \pi \rightarrow \pi,\ -\pi$ } ← ۲ جواب دارد

$$۲\sin(x)\cos ۲x + \sin x = ۱$$

$$۲\left(\frac{۱}{۲}\left(\sin(۳x) + \sin(-x)\right)\right) + \sin x = ۱$$

$$\sin ۳x \cancel{- \sin x} \cancel{+ \sin x} = \cancel{۱} \;\; ۲k\pi + \frac{\pi}{۲}$$

$$۳x = \frac{۲k\pi}{۳} + \frac{\pi}{۶}$$

۲

$$\frac{\pi}{۶} \qquad \frac{۵\pi}{۶} \qquad \frac{۹\pi}{۶}$$

$$\frac{۱۵\pi}{۶} = \frac{\cancel{۴}\,۵\pi}{۲}$$

۴

$$\frac{۱}{\cos ۲x} + \frac{۱}{-\sin ۴x} = ۰$$

$$\cos ۲x(-۲\sin ۲x + ۱)$$

$$\frac{-\sin ۴x + \cos ۲x}{-\cos ۲x \sin ۴x} = ۰$$

۲

$$\sin ۲x = \frac{۱}{۲} \quad \longleftarrow \quad -۲\sin ۲x + ۱ = ۰ \quad \longleftarrow \quad \cos ۲x \neq ۰$$

جواب نمی‌کنند $\frac{\pi}{۱۲}$ و $\frac{۵\pi}{۱۲}$ $\Rightarrow \alpha = \frac{۴\pi}{۳}$ $\quad \cancel{\tan}\ \frac{۲\pi}{۳} = -\sqrt{۳}$

۵ـ

$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ ۱,۵

$m(\cos x-\sin\alpha)$

$\sin\left(x-\frac{\pi}{4}\right)$

$-m\left(\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)\right)-3\sqrt{6}\sin 2x=\sqrt{6}$

$\sin\left(x+\frac{\pi}{4}-\frac{\pi}{2}\right)$

$\cos\alpha=\frac{1}{\sqrt{2}}$

$\frac{\cancel{\sqrt{2}}m}{\sqrt{3}}=\frac{\cancel{\sqrt{6}}}{\sqrt{3}}(3\sin 2a-1)\Rightarrow \boxed{m=9\sin^2 a-3}$

۶ـ

$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x+\frac{\pi}{6}-\frac{\pi}{2}\right)=1$ ۱,۵

$\sin^2\left(\frac{\pi}{6}+x\right)=1$ $\quad \frac{\pi}{6}+x=k\pi\pm\frac{\pi}{2}$

$x=k\pi\mp\frac{4\pi}{6}$

$x=k\pi\pm\frac{2\pi}{6}$

۷ـ

$\sin x+\sqrt{3}\cos x=\sqrt{2}$ ۱,۲۵

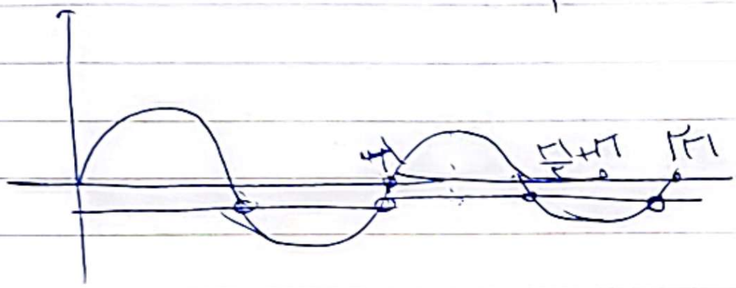
$2\sin\left(x+\underbrace{\cancel{\arctan\sqrt{3}}}_{\frac{\pi}{3}}\right)=\sqrt{2}\Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\frac{\sqrt{2}}{2}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\Rightarrow x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4}\Rightarrow x=2k\pi+\frac{5\pi}{12}$

$4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1$ ۱-

$\sin 2x = \frac{1}{2}$



۲

$\frac{2\cancel{4}\times\left(\frac{4\pi+\pi}{2}\right)}{\cancel{4}} \Rightarrow 4\pi+\pi = \underline{5\pi}$

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha) = \frac{1}{8}$ ۱,۵ ۹-

$8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8} \Rightarrow \cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64}$

$\sin\alpha(\cos\alpha\cos 2\alpha\cos 4\alpha = \frac{1}{8})$

$\frac{1}{2}\sin 2\alpha\cos 2\alpha = \frac{1}{4}\sin 4\alpha \Rightarrow \frac{1}{8}\sin 8\alpha = \frac{1}{8}\sin\alpha \Rightarrow$

$4\pi+4\pi - 8\alpha = \alpha = 2k\pi \quad 2k\pi + 8\alpha = \alpha \Rightarrow \quad \alpha = \frac{2k\pi}{7}$ یا $\frac{2k\pi}{9}$

$\tan(3x) = \frac{1}{\tan x} \quad \cot x \quad \tan\left(\frac{\pi}{2}-x\right)$ ۱۵

۱,۲۵

$k\pi + 3x = \frac{\pi}{2} - x \Rightarrow k\pi - \frac{\pi}{2} = -4x \Rightarrow$

$\frac{k\pi - \frac{\pi}{2}}{-4} = \boxed{\frac{k\pi}{4} + \frac{\pi}{8}}$

۵

$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt2}{2}(\cos x-\sin x) = \frac{\sqrt3}{3} \rightarrow \cos x-\sin x = \frac{\frac{\sqrt3}{3}}{\frac{1}{\sqrt2}}$$

$$\cos x-\sin x = \frac{\sqrt2}{\sqrt3} \xrightarrow{\text{گویا}} \frac{\sqrt6}{3} \rightarrow \boxed{\cos x-\sin x=\frac{\sqrt6}{3}}$$

$$(\cos x-\sin x)^2 = 1-\sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x-\sin x) - 3\sqrt6(\sin 2x) = \sqrt6 \rightarrow \frac{\sqrt6}{3}m - 3\sqrt6(\frac{1}{3}) = \sqrt6$$

$$\frac{\sqrt6}{3}m - \sqrt6 = \sqrt6 \rightarrow \frac{m\sqrt6}{3} = 2\sqrt6 \rightarrow \boxed{m=6}$$

۶

$$\cos(x-\frac{\pi}{3}) = \cos(\frac{\pi}{3}-x)$$

$$\frac{\pi}{3}-x+x+\frac{\pi}{6} = \frac{\pi}{2} \rightarrow \cos(\frac{\pi}{3}-x) = \sin(x+\frac{\pi}{6})$$

$$\sin^2(x+\frac{\pi}{6}) = 1 \rightarrow \sin(x+\frac{\pi}{6}) = \pm1 \rightarrow x+\frac{\pi}{6} = k\pi+\frac{\pi}{2}$$

$$x = k\pi+\frac{\pi}{3} \xrightarrow{0\le x\le 2\pi} k=0 \rightarrow x=\frac{\pi}{3} \quad , \quad k=1 , x=\frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

۷

عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt3}{2}\cos x = \frac{\sqrt2}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt2}{2} \rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt2}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi-\frac{\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0,1 \rightarrow x=-\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \rightarrow x = 2k\pi+\frac{5\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0 \rightarrow x=\frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

(۹)

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \ast$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAX } A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\text{MAX } A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\text{MAX } A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!

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(۱۰)

$$\tan 3x = \frac{1}{\tan x} = \cot x \rightarrow \tan 3x = \tan\left(\frac{\pi}{2}-x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \rightarrow 4x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$