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①

$8\cos x = 1+\tan^2 x \to 8\cos x = \frac{1}{\cos^2 x} \to \cos x = \frac{1}{2} \to \cos x = \cos\frac{\pi}{3} \to x = 2k\pi \pm \frac{\pi}{3}$

$\xrightarrow{[0,2\pi]} x = \frac{\pi}{3}, \frac{5\pi}{3} \to$ دو جواب

②

$5\left(\frac{1-\cos 2x}{2}\right) + 2\cos 3x = -2 \to \frac{5}{2} - \frac{5}{2}\cos 2x + 2\cos 3x = -2 \to \underbrace{4\cos 3x}_{R=[-4,4]} - \underbrace{5\cos 2x}_{R=[-5,5]} = -9$

$\cos 2x = -1 \to 3x = 2k\pi+\pi \to x = \frac{2k\pi+\pi}{3}$

$\cos 3x = 1 \to 2x = 2k\pi \to x = 2k\pi$

$\xrightarrow{[-\pi,\pi]} x = \{-\pi, 0, \pi\} \to$ ۳ جواب

③

$\sin x(2\cos 2x+1) = 1 \to \sin x(2(1-2\sin^2 x)+1) = 1 \to \sin x(3-4\sin^2 x) = 1$

$\to \underbrace{3\sin x - 4\sin^3 x}_{\sin 3x} = 1 \to \sin 3x = \sin\frac{\pi}{2} \to x = \frac{2k\pi}{3}+\frac{\pi}{6} \xrightarrow{x\in[0,2\pi]} x = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\} \xrightarrow{\text{جمع}} \boxed{\frac{5\pi}{2}}$

④

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \to \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \to \cos 2x = \sin 4x$

$\to \cos 2x(\sin 2x - 1) = 0 \to \begin{cases} \cos 2x = 0 \to x = k\pi+\frac{\pi}{4}, k\pi+\frac{3\pi}{4} \\ \sin 2x = 1 \to x = k\pi+\frac{\pi}{4} \end{cases} \to x = \{\frac{\pi}{4}, \frac{3\pi}{4}\}$

$\xrightarrow{\text{اشتراک}} \frac{\pi}{2} \to \tan(2\times\frac{\pi}{2}) = \tan\pi = \boxed{0}$

⑤

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{2}}{3} \to \cos x - \sin x = \frac{\sqrt{6}}{3} \to (\cos x - \sin x)^2 = 1 - \sin 2x \to \sin 2x = \frac{1}{3}$

$\to m(\frac{\sqrt{6}}{3}) = 3\sqrt{6}\sin(2x) + \sqrt{6} \xrightarrow{\div\sqrt{6}} m = 9(\frac{1}{3}) + 3 = \boxed{6}$

⑥

$-1 \le \sin x \le 1 \to \sin(x+\frac{\pi}{6}) = 1 \to x = 2k\pi+\frac{\pi}{3}$

$-1 \le \cos x \le 1 \to \cos(x-\frac{\pi}{3}) = 1 \to x = 2k\pi+\frac{\pi}{3}$

$\xrightarrow{\cap} x = \{\frac{\pi}{3}\} \to$ جواب ①

⑦

$\sqrt{3}\cos x + \sin x = 2\sin(x+\frac{\pi}{3}) = \sqrt{3} \to \sin(x+\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \to x = 2k\pi+\frac{\pi}{3}$, $x = 2k\pi$

$\to x = \{0, 2\pi, \frac{\pi}{3}\} \xrightarrow{\text{جمع}} \boxed{\frac{7\pi}{3}}$

⑧

$-4\sin x\cos x = -2\sin 2x \to \sin 2x = \frac{-1}{2} \to \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi+\pi+\frac{\pi}{6} \to x = k\pi + \frac{7\pi}{12} \end{cases}$

$\xrightarrow{[0,2\pi]} x = \{\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}\} \xrightarrow{\text{جمع}} \boxed{5\pi}$

⑨

$2\cos^2 x \cdot 2\cos^2 2x \cdot 2\cos^2 4x = \frac{1}{8} \to 8\cos^2 x \cdot \cos^2 2x \cdot \cos^2 4x = \frac{1}{8} \to |\cos x \cdot \cos 2x \cdot \cos 4x| = \frac{1}{8}$

جایگذاری گزینه‌ها $x = \frac{\pi}{3} \to |\cos x \cdot \cos 2x \cdot \cos 4x| = |\frac{1}{2} \cdot \frac{-1}{2} \cdot \frac{-1}{2}| = \frac{1}{8}$ ✓

$B = \frac{2\pi}{3} \to |\frac{-1}{2} \cdot \frac{-1}{2} \cdot \frac{-1}{2}| = \frac{1}{8}$ ✓

$\to A = \{\frac{\pi}{3}, \frac{2\pi}{3}\}$

$\max_A = \boxed{\frac{2\pi}{3}}$

۱۰

$\tan\alpha \cdot \tan 3\alpha = 1 \xrightarrow{\div \tan\alpha} \cot\alpha = \tan 3\alpha \longrightarrow \tan 3\alpha = \tan\left(\frac{\pi}{2} - x\right)$

$\longrightarrow 3x = k\pi + \frac{\pi}{2} - x \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8} \xrightarrow{[\pi, 2\pi]} x = \left\{\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\} \xrightarrow{\text{جمع}} \boxed{6\pi}$