نام : دنیا مهدی   شماره تکلیف : ۱۷   [illegible] ۱ دوازدهم دختران B   ۱۸

①
$8\cos x = 1+\tan^2 x \to 8\cos x = \frac{1}{\cos^2 x} \to \cos x = \frac{1}{2} \to \cos x = \cos\frac{\pi}{3} \to x = 2k\pi \pm \frac{\pi}{3}$

$\xrightarrow{[0,2\pi]} x = \frac{\pi}{3}, \frac{5\pi}{3} \to$ دو جواب

۲

②
$5\left(\frac{1-\cos 2x}{2}\right) + 2\cos 3x = -2 \to \frac{5}{2} - \frac{5}{2}\cos 2x + 2\cos 3x = -2 \to 4\cos 3x - 5\cos 2x = -9$

باید بین جواب های ۲ بازه اشتراک بگیری $R=[-4,4]$ $R=[-5,5]$

$\cos 3x = -1 \to 3x = 2k\pi+\pi \to x = \frac{2k\pi+\pi}{3}$

$\cos 2x = 1 \to 2x = 2k\pi \to x = 2k\pi$

$\xrightarrow{[-\pi,\pi]} x = \{-\pi, \cancel{0}, \pi\} \to$ ۳ جواب

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③
$\sin x(2\cos 2x+1) = 1 \to \sin x(2(1-2\sin^2 x)+1) = 1 \to \sin x(3-4\sin^2 x) = 1$

$\to \underbrace{3\sin x - 4\sin^3 x}_{\sin 3x} = 1 \to \sin 3x = \sin\frac{\pi}{2} \to x = \frac{2k\pi}{3}+\frac{\pi}{6} \xrightarrow{x\in[0,2\pi]} x = \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\} \xrightarrow{\text{جمع}} \boxed{\frac{5\pi}{2}}$

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④
$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \to \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \to \cos 2x = \sin 4x$

$\to \cos 2x(\sin 2x - 1) = 0 \to \begin{cases} \cos 2x = 0 \to x = k\pi+\frac{\pi}{4}, k\pi+\frac{3\pi}{4} \\ \sin 2x = 1 \to x = k\pi+\frac{\pi}{4} \end{cases} \to x = \{\frac{\pi}{4}, \frac{3\pi}{4}\}$

$\xrightarrow{\text{آسان}} \frac{\pi}{2} \to \tan(2\times\frac{\pi}{2}) = \tan\pi = \boxed{0}$

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⑤
$\cos(x+\frac{\pi}{4}) = \frac{\sqrt2}{2}(\cos x - \sin x) = \frac{\sqrt3}{3} \to \cos x - \sin x = \frac{\sqrt6}{3} \to (\cos x - \sin x)^2 = 1 - \sin 2x \to \sin 2x = \frac{1}{3}$

$\to m(\frac{\sqrt6}{3}) = 3\sqrt6\sin(2x) + \sqrt6 \xrightarrow{\div\sqrt6} m = 9(\frac{1}{3}) + 3 = \boxed{6}$

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⑥
$-1 \le \sin x \le 1 \to \sin(x+\frac{\pi}{6}) = 1 \to x = 2k\pi+\frac{\pi}{3}$

$-1 \le \cos x \le 1 \to \cos(x-\frac{\pi}{3}) = 1 \to x = 2k\pi+\frac{\pi}{3}$

$\xrightarrow{\cap} x = \{\frac{\pi}{3}\} \to$ ① جواب   و $\frac{4\pi}{3}$

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⑦
$\sqrt3\cos x + \sin x = 2\sin(x+\frac{\pi}{3}) = \sqrt3 \to \sin(x+\frac{\pi}{3}) = \frac{\sqrt3}{2} \to x = 2k\pi+\frac{\pi}{3}$, $x = 2k\pi$

$\to x = \{0, 2\pi, \frac{\pi}{3}\} \xrightarrow{\text{جمع}} \boxed{\frac{7\pi}{3}}$

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⑧
$-4\sin x\cos x = -2\sin 2x \to \sin 2x = \frac{-1}{2} \to \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi + \pi + \frac{\pi}{6} \to x = k\pi + \frac{7\pi}{12} \end{cases}$

$\xrightarrow{[0,2\pi]} x = \{\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}\} \xrightarrow{\text{جمع}} \boxed{5\pi}$

۲

⑨
$2\cos^2 x \cdot 2\cos^2 2x \cdot 2\cos^2 4x = \frac{1}{8} \to 8\cos^2 x\cdot\cos^2 2x\cdot\cos^2 4x = \frac{1}{8} \to |\cos x\cdot\cos 2x\cdot\cos 4x| = \frac{1}{8}$

[illegible]
$x = \frac{\pi}{3} \to |\cos x\cdot\cos 2x\cdot\cos 4x| = |\frac{1}{2}\cdot\frac{-1}{2}\cdot\frac{-1}{2}| = \frac{1}{8}$ ✓

$B = \frac{2\pi}{3} \to |\frac{-1}{2}\cdot\frac{-1}{2}\cdot\frac{-1}{2}| = \frac{1}{8}$ ✓

$\to A = \{\frac{\pi}{3}, \frac{2\pi}{3}\}$

$\max_A = \boxed{\frac{2\pi}{3}}$

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۱۰

۲

$$\tan\alpha \cdot \tan ۳\alpha = ۱ \xrightarrow{\div \tan\alpha} \cot\alpha = \tan ۳\alpha \longrightarrow \tan ۳\alpha = \tan\left(\frac{\pi}{۲} - \alpha\right)$$

$$\longrightarrow ۳\alpha = k\pi + \frac{\pi}{۲} - \alpha \longrightarrow \alpha = \frac{k\pi}{۴} + \frac{\pi}{۸} \xrightarrow{[\pi, ۲\pi]} \alpha = \left\{ \frac{۹\pi}{۸}, \frac{۱۱\pi}{۸}, \frac{۱۳\pi}{۸}, \frac{۱۵\pi}{۸} \right\} \xrightarrow{\text{جمع}} \boxed{۶\pi}$$

(۳) -

$\cos 3x = 2\cos^2 \frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2(2\cos^2 \frac{3x}{2} - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2 \frac{3x}{2} \cancel{-2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2 \frac{3x}{2} = 0 \rightarrow \cos \frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \le x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

$\boxed{\text{اشتراک دو مجموعه جواب } \{-\pi, \pi\} \text{ است}}$

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(۴)

$$\begin{cases} \sin(2x + \frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2} + 4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$\cos 2x - 2\sin 2x \cos 2x = 0 \rightarrow \cos 2x(1 - 2\sin 2x) = 0$

$\rightarrow \cos 2x = 0$ ✗

$\rightarrow \sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$ یا $k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

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(۶)

$\cos(x - \frac{\pi}{3}) = \cos(\frac{\pi}{3} - x)$

$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos(\frac{\pi}{3} - x) = \sin(x + \frac{\pi}{6})$

$\sin^2(x + \frac{\pi}{6}) = 1 \rightarrow \sin(x + \frac{\pi}{6}) = \pm 1 \rightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k = 0 \rightarrow x = \frac{\pi}{3}$ ، $k = 1$ ، $x = \frac{4\pi}{3}$

$\boxed{\text{معادله ۲ جواب در بازه دارد}}$

⑦ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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⑨

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \to 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha^{*}}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \to \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \to \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{ یا } \ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{ یا } \ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \to x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \to x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \to x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \to x = \frac{7\pi}{9}$$

$$\to \boxed{\max A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!