نام و نام خانوادگی: آدرینا حنیدی — پاسخ شماره تکلیف: ۱۷ — کلاس: ۱۲B

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$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

$x = 2k\pi \pm \frac{\pi}{3}$ $\quad x = \frac{\pi}{3}, \frac{5\pi}{3}$

دو جواب

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$5\sin^2(x) + 2\cos(3x) = -2 \qquad [-\pi, \pi]$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\Rightarrow (\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0 \quad \Delta < 0$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi \Rightarrow x = \pi, -\pi$ دو جواب

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$2\sin x \cos 2x + \sin x = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$4\sin^3 x - 3\sin x + 1 = 0 \Rightarrow 4\sin^3 x - 4\sin x + \sin x + 1 = 0$

$\Rightarrow (\sin x + 1)(4\sin^2 x - 4\sin x + 1) = 0 \quad (2\sin x - 1)^2$

$\sin x = -1 \Rightarrow x = 2k\pi + \frac{3\pi}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6}$, $x = 2k\pi + \frac{5\pi}{6}$

$S = \frac{5\pi}{2}$ مجموع

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

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$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \Rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0$

$\frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = 0$

$\cos 2x \neq 0 \Rightarrow 2x \neq k\pi + \frac{\pi}{2} \Rightarrow x \neq \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 4x \neq 0 \Rightarrow 4x \neq k\pi \Rightarrow x \neq \frac{k\pi}{4}$

$\sin 4x = \cos 2x, \ \sin(\frac{\pi}{2} - 2x)$

$4x = 2k\pi + \frac{\pi}{2} - 2x \Rightarrow 6x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12}$

$4x = 2k\pi + \pi - \frac{\pi}{2} + 2x \Rightarrow 2x = 2k\pi + \frac{\pi}{2} \Rightarrow x = k\pi + \frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12} \quad \alpha = \frac{\pi}{3} \Rightarrow 2\alpha = \frac{2\pi}{3} \quad \tan\frac{2\pi}{3} = -\sqrt{3}$

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$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

$\sin(\frac{\pi}{4} - x) = \frac{1}{\sqrt{3}}$

$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{\sqrt{6}}{3}$

توان ۲ $\Rightarrow 1 - \sin 2x = \frac{2}{3} \Rightarrow \sin 2x = \frac{1}{3}$

$m(\frac{\sqrt{6}}{3}) - \sqrt{6} = \sqrt{6}$

$m = 6$

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$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1$, $\quad \cos(x-\frac{\pi}{3}) = \cos(\frac{\pi}{3}-x)$

$\sin^2(x+\frac{\pi}{6})=1 \rightarrow \begin{cases} \sin(x+\frac{\pi}{6})=1 & x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \Rightarrow x=2k\pi+\frac{\pi}{3} \\ \sin(x+\frac{\pi}{6})=-1 & x+\frac{\pi}{6}=2k\pi+\frac{3\pi}{2} \Rightarrow x=2k\pi+\frac{4\pi}{3} \end{cases}$

$\Rightarrow x=k\pi+\frac{\pi}{3}$

در جواب $\boxed{x=\frac{\pi}{3}, \frac{4\pi}{3}}$

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$\sin x+\sqrt{3}\cos x=\sqrt{2}$

$\sin x+\tan\frac{\pi}{3}\cos x=\sqrt{2}$

$\frac{\sin(x+\frac{\pi}{3})}{\cos\frac{\pi}{3}}=\sqrt{2} \Rightarrow \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12}$, $\quad -\pi\le x\le 2\pi \Rightarrow -\pi\le 2k\pi-\frac{\pi}{12}\le 2\pi$

$\Rightarrow -\frac{11}{24}\le k\le\frac{25}{24} \Rightarrow k=0,1$

$x=-\frac{\pi}{12}, \frac{23\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12} \overset{k=0}{\Rightarrow} x=\frac{5\pi}{12}$

$\boxed{S=-\frac{\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}=\frac{27\pi}{12}}$

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$4\sin x\sin(\frac{3\pi}{2}-x)=1 \qquad [0,2\pi]$

$-4\sin x\cos x=1$

$-2\sin 2x=1 \Rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \Rightarrow x=\frac{11\pi}{12}, 2\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12} \Rightarrow x=\frac{7\pi}{12}, \pi+\frac{7\pi}{12}$

$\boxed{S=2\pi+2\pi+\pi=5\pi}$

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$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$

$\frac{\sin\alpha}{\sin\alpha}\times\cos\alpha\cos 2\alpha\cos 4\alpha=\frac{1}{8}$

$\frac{1}{2}\sin 2\alpha$, $\quad \frac{1}{4}\sin 4\alpha$

$\frac{\frac{1}{8}\sin 8\alpha}{\sin\alpha}=\frac{1}{8} \Rightarrow \sin 8\alpha=\sin\alpha$

$\boxed{\alpha_{max}=\pi}$

$8\alpha=2k\pi+\alpha$

$\alpha=\frac{2k\pi}{7}$

$8\alpha=2k\pi+\pi-\alpha$

$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$

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$\tan 3x\tan x=1 \Rightarrow \tan x=\cot 3x \Rightarrow \tan x=\tan(\frac{\pi}{2}-3x)$

$x=k\pi+\frac{\pi}{2}-3x \Rightarrow 4x=k\pi+\frac{\pi}{2} \Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$

$\pi\le x\le 2\pi \Rightarrow \pi\le\frac{k\pi}{4}+\frac{\pi}{8}\le 2\pi$

$1\le\frac{k}{4}+\frac{1}{8}\le 2 \Rightarrow \frac{7}{2}\le k\le\frac{15}{2}$

$x=\pi+\frac{\pi}{8}, \frac{5\pi}{4}+\frac{\pi}{8}, \frac{3\pi}{2}+\frac{\pi}{8}, \frac{7\pi}{4}+\frac{\pi}{8} \qquad k=4,5,6,7$

$\boxed{S=6\pi}$