نام و نام خانوادگی: آدرینا حمیدی   پاسخ شماره تکلیف: ۱۷   کلاس: ۱۲B

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$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x = \cancel{1+\tan^2 x} \ \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2}$

۱۹/۵ آفرین

$x = 2k\pi \pm \frac{\pi}{3}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

دو جواب

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$5\sin^2(x) + 2\cos(3x) = -2$ $[-\pi, \pi]$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\Rightarrow (\cos x + 1)(\underbrace{8\cos^2 x - 13\cos x + 7}_{\Delta<0}) = 0$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi \Rightarrow x = \pi, -\pi$ دو جواب

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$2\sin x\cos 2x + \sin x = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$4\sin^3 x - 3\sin x + 1 = 0 \Rightarrow 4\sin^3 x - 4\sin x + \sin x + 1 = 0$

$\Rightarrow (\sin x + 1)(\underbrace{4\sin^2 x - 4\sin x + 1}_{(2\sin x - 1)^2}) = 0$

$\sin x = -1 \Rightarrow x = 2k\pi + \frac{3\pi}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \frac{5\pi}{6}$

مجموع $\boxed{S = \frac{5\pi}{2}}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

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$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \Rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0$

$\frac{\sin 4x - \cos 2x}{\cos 2x \sin 4x} = 0$

$\sin 4x = \cos 2x$, $\sin(\frac{\pi}{2} - 2x)$

$4x = 2k\pi + \frac{\pi}{2} - 2x \Rightarrow 6x = 2k\pi + \frac{\pi}{2}$

$\Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12}$

$4x = 2k\pi + \pi - \frac{\pi}{2} + 2x \Rightarrow 2x = 2k\pi + \frac{\pi}{2}$

$\Rightarrow x = k\pi + \frac{\pi}{4}$

$\cos 2x \neq 0 \Rightarrow 2x \neq k\pi + \frac{\pi}{2} \Rightarrow x \neq \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 4x \neq 0 \Rightarrow 4x \neq k\pi \Rightarrow x \neq \frac{k\pi}{4}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ $\alpha = \frac{\pi}{3} \Rightarrow 2\alpha = \frac{2\pi}{3}$ $\tan\frac{2\pi}{3} = -\sqrt{3}$

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$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$

$m(\frac{\sqrt{6}}{3}) - \sqrt{6} = \sqrt{6}$

$\boxed{m = 6}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

$\sin(\frac{\pi}{4} - x) = \frac{1}{\sqrt{3}}$

$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{\sqrt{6}}{3}$

توان ۲ $\Rightarrow 1 - \sin 2x = \frac{2}{3} \Rightarrow \sin 2x = \frac{1}{3}$

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$$\sin(x+\frac{\pi}{6})\underbrace{\cos(x-\frac{\pi}{3})}_{\cos(\frac{\pi}{3}-x)}=1$$ ۶

$$\sin^2(x+\frac{\pi}{6})=1 \to \begin{cases}\sin(x+\frac{\pi}{6})=1 & x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \Rightarrow x=2k\pi+\frac{\pi}{3}\\ \sin(x+\frac{\pi}{6})=-1 & x+\frac{\pi}{6}=2k\pi+\frac{3\pi}{2} \Rightarrow x=2k\pi+\frac{4\pi}{3}\end{cases}$$

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$$\Rightarrow x=k\pi+\frac{\pi}{3}$$

در جواب $\boxed{x=\frac{\pi}{3}, \frac{4\pi}{3}}$

$$\sin x+\sqrt{3}\cos x=\sqrt{2}$$ ۷

$$\sin x+\tan\frac{\pi}{3}\cos x=\sqrt{2}$$

$$\frac{\sin(x+\frac{\pi}{3})}{\cos\frac{\pi}{3}}=\sqrt{2} \Rightarrow \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$$

$$x=-\frac{\pi}{12}, \frac{23\pi}{12}$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12} \quad -\pi\le x\le 2\pi \Rightarrow -\pi\le 2k\pi-\frac{\pi}{12}\le 2\pi$$

$$\Rightarrow -\frac{11}{24}\le k\le\frac{25}{24} \Rightarrow k=0,1$$ ۲

$$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12} \xRightarrow{k=0} x=\frac{5\pi}{12}$$

$$\boxed{S=-\frac{\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}=\frac{27\pi}{12}}$$

$$4\sin x\sin(\frac{3\pi}{2}-x)=1 \qquad [0,2\pi]$$ ۸

$$-4\sin x\cos x=1$$

$$-2\sin 2x=1 \Rightarrow \sin 2x=-\frac{1}{2}$$

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$$2x=2k\pi-\frac{\pi}{6} \Rightarrow x=k\pi-\frac{\pi}{12} \Rightarrow x=\frac{11\pi}{12}, 2\pi-\frac{\pi}{12}$$

$$2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=k\pi+\frac{7\pi}{12} \Rightarrow x=\frac{7\pi}{12}, \pi+\frac{7\pi}{12}$$

$$\boxed{S=\cancel{2\pi}+2\pi+\pi=5\pi}$$

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$$ ۹

$$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$$

$$\frac{\sin\alpha}{\sin\alpha}\times\cos\alpha\cos 2\alpha\cos 4\alpha=\frac{1}{8}$$

$$\frac{1}{2}\sin 2\alpha \qquad \frac{1}{4}\sin 4\alpha \qquad \frac{\frac{1}{8}\sin 8\alpha}{\sin\alpha}=\frac{1}{8} \Rightarrow \sin 8\alpha=\sin\alpha$$

$$\boxed{\alpha_{max}=\pi}$$

$$8\alpha=2k\pi+\alpha$$

$$\alpha=\frac{2k\pi}{7}$$

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$$8\alpha=2k\pi+\pi-\alpha$$

$$\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$$

$$\tan 3x\tan x=1 \Rightarrow \tan x=\cot 3x \Rightarrow \tan x=\tan(\frac{\pi}{2}-3x)$$ ۱۰

$$x=k\pi+\frac{\pi}{2}-3x \Rightarrow 4x=k\pi+\frac{\pi}{2} \Rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$$ ۲

$$\pi\le x\le 2\pi \Rightarrow \pi\le\frac{k\pi}{4}+\frac{\pi}{8}\le 2\pi$$

$$1\le\frac{k}{4}+\frac{1}{8}\le 2 \Rightarrow \frac{7}{2}\le k\le\frac{15}{2}$$

$$x=\pi+\frac{\pi}{8}, \frac{5\pi}{4}+\frac{\pi}{8}, \frac{3\pi}{2}+\frac{\pi}{8}, \frac{7\pi}{4}+\frac{\pi}{8} \qquad k=4,5,6,7$$

$$\boxed{S=6\pi}$$

$$1+\cos 2x = 2\cos^2 x$$

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha\neq 0]{\times\sin\alpha \ ★}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!