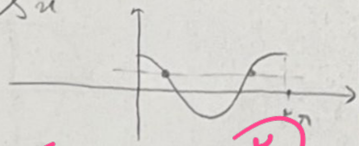
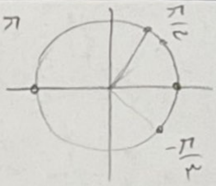
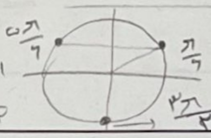
۱۸

نام و نام خانوادگی ....علیرضا پوربهیری.... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ....دوازدهم....

$8\cos x = 1+\tan^2 x \Rightarrow \frac{1}{\cos^2 x} \Rightarrow 8\cos x = \frac{1}{\cos^2 x}$ $8\cos^3 x = 1 \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2}$  ۲ جواب در بازه‌ی $(0, 2\pi]$ ۲	۱
$5\sin^2 x + 2\cos 4x + 2 = 0$ $0 \le 5\sin^2 x \le 5 \quad -2 \le 2\cos 4x \le 2$ $0 - 2 + 2 = 0$ $\sin^2 x = 0 \to x = 0, \pi$ $\cos 4x = -1$ $4x = 2k\pi \pm \pi$ $x = \frac{2k\pi}{4} \pm \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}, -\frac{\pi}{4}$  $\{0, \pi\} \cap \{-\frac{\pi}{4}, \frac{\pi}{4}\} = \emptyset$ در این بازه جواب ندارد ۱/۵	۲
$2\sin x \cdot \cos 2x + \sin x = 1$ $(1 - 2\sin^2 x)$ $3\sin x - 4\sin^3 x - 1 = 0$ ضریب توان های زوج و فرد برابر $\Rightarrow (\sin x + 1)(2\sin x - 1)^2 \Rightarrow \sin x = -1, \ \sin x = \frac{1}{2}$ $4\sin^3 x - 3\sin x + 1 \ \vert \ \sin x + 1$ $-4\sin^3 x \mp 4\sin^2 x \quad 4\sin^2 x - 4\sin x + 1$ $-4\sin^2 x - 4\sin x$ $-4\sin^2 x \mp 4\sin x$  جواب $\Rightarrow \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$ ۲	۳
$\frac{1}{\sin(\frac{\pi}{2} + 2x)} + \frac{1}{\cos(\frac{\pi}{2} + 4x)} = \frac{-\sin 4x + \cos 2x}{-2\sin 2x \cos^2 2x} \Rightarrow -2\sin 2x\cos 2x + \cos 2x$ $\cos 2x \quad \sin(-4x) \quad -2\sin 2x\cos 2x$ $\tan\frac{2\pi}{3} = -\sqrt{3}$ جواب $\Rightarrow \frac{\cos 2x(1 - 2\sin 2x)}{-2\sin 2x\cos 2x} = 0 \to \frac{1}{2} = \sin 2x, \ 2x = 2k\pi + \frac{\pi}{6} \to \frac{\pi}{12}$ $2x = 2k\pi + \frac{5\pi}{6} \to \frac{5\pi}{12} \Rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$ ۳	۴
$m(\sqrt{2}\sin(\frac{\pi}{4} - x)) - 3\sqrt{6}\cos(\frac{\pi}{2} - 2x) \Rightarrow (2\cos^2(\frac{\pi}{4} - x) - 1)$ $\sqrt{2}m\cos(\frac{\pi}{4} + x) - 3\sqrt{6}\sin 2x = \sqrt{6} \quad (2\sin^2(\frac{\pi}{4} + x) - 1)$ $\frac{\sqrt{2}m}{\sqrt{3}} - 6\sqrt{6}\sin^2(\frac{\pi}{4} + x) - 3\sqrt{6} = \sqrt{6} \Rightarrow \frac{\sqrt{2}m}{\sqrt{3}} - 4\sqrt{6} = 4\sqrt{6}$ $\frac{\sqrt{2}m}{\sqrt{3}} = 8\sqrt{6}$ $m = 24$ ۱/۵	۵

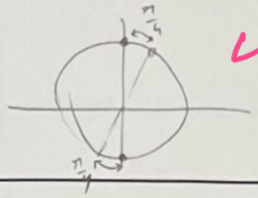
۶ ✓ جواب

$\sin(x+\frac{\pi}{4})\cos(\frac{\pi}{2}-x)=1$

$\sin(x+\frac{\pi}{4})$

$\sin^2(x+\frac{\pi}{4})=1$

$x+\frac{\pi}{4}=\frac{\pi}{2}$



۷ جواب

$\sin x+\sqrt{3}\cos x=\sqrt{2}$ $\quad \tan\frac{\pi}{3}=\sqrt{3}$ $\quad [-4\pi,4\pi]$

$2\sin(x+\frac{\pi}{3})=\sqrt{2}\rightarrow \sin x+\frac{\pi}{3}=\frac{\pi}{4}$

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\rightarrow x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4}\rightarrow x=2k\pi+\frac{5\pi}{12}$

k	0	1
x	$-\frac{\pi}{12}$	$\frac{5\pi}{12}$

$\frac{3\pi}{4}+4\pi=\frac{19\pi}{4}$, $-\frac{\pi}{12}+4\pi=\frac{47\pi}{12}$, $\frac{5\pi}{12}+4\pi=\frac{53\pi}{12}$

جواب $\frac{5\pi}{12}$

۸

$4\sin x\cos(\frac{\pi}{2}-\frac{\pi}{3}+x)\Rightarrow 4\sin x\cdot\cos(x-\frac{\pi}{3})\Rightarrow$

$\frac{\pi}{3}-x=\frac{\pi}{3}-x$

جواب $\frac{\pi}{3}$ ✓

$-4\sin x\cos x=1$

$-2\sin 2x=1\rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6}\rightarrow x=k\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{7\pi}{6}\rightarrow x=k\pi+\frac{7\pi}{12}$

k	1	2
x	$\frac{11\pi}{12}$	$\frac{19\pi}{12}$

$\frac{11\pi}{12}-2\pi=-\frac{13\pi}{12}$, $\frac{19\pi}{12}-2\pi=-\frac{5\pi}{12}$, $\frac{11\pi}{12}-\pi=-\frac{\pi}{12}$, $\frac{19\pi}{12}-\pi=\frac{7\pi}{12}$

$\frac{11\pi}{12}$, $\frac{19\pi}{12}$

۹

$\cos^2\alpha\cdot\cos^2 2\alpha\cdot\cos^2 4\alpha=\frac{1}{64}$

$\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha=\frac{1}{8}$ $\quad \frac{\sin 8\alpha}{8\sin\alpha}$

$\frac{1}{2}\sin 2\alpha\rightarrow\frac{1}{4}\sin 4\alpha\rightarrow\frac{1}{8}\sin 8\alpha$

$\Rightarrow\frac{\sin 8\alpha}{8\sin\alpha}=1\xrightarrow{\sin\alpha\neq 0}\sin 8\alpha=8\sin\alpha$

$8\alpha=2k\pi+\alpha\Rightarrow\frac{2k\pi}{7}$

$8\alpha=2k\pi+\pi-\alpha\Rightarrow\frac{2k\pi}{9}+\frac{\pi}{9}$

$(0,\pi)\rightarrow k=4\rightarrow\frac{8\pi}{9}+\frac{\pi}{9}\Rightarrow\alpha_{max}$ جواب $\frac{\pi}{3}$

۱۰

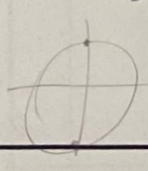
$\frac{\sin 2x}{\cos 2x}\times\frac{\sin x}{\cos x}=1\xrightarrow{\cos 2x\cos x\neq 0}\cos 2x\cos x-\sin 2x\sin x=0$

$\Rightarrow\cos 3x=0$ $\quad 3x=2k\pi\pm\frac{\pi}{2}\rightarrow x=\frac{2k\pi}{3}\pm\frac{\pi}{6}$

$[\pi,2\pi]$

k	2	3
x	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$

$\frac{7\pi}{6}-2\pi=-\frac{5\pi}{6}$



$x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

جواب 3

۲)

$\cos 2x = 2\cos^2 x - 1$ (فرمول ضای)

$5\sin^2 x + 2(2\cos^2 x - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2 x - 2 = -2$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{0 \le x < \pi} x = 0, \pi$

$4\cos^2 \frac{3x}{2} = 0 \rightarrow \cos \frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{0 \le x < 2\pi} x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

اشتراک دو مجموعه جواب $\{\pi, \frac{\pi}{3}\}$ است

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۵)

$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{2} \rightarrow \cos x - \sin x = \frac{\sqrt{6}}{2}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \rightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \rightarrow \boxed{\sin 2x = \frac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \rightarrow \frac{\sqrt{6}}{3} m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$

$\frac{\sqrt{6}}{3} m - \sqrt{6} = \sqrt{6} \rightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \rightarrow \boxed{m = 6}$

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۱)

$\sin(\frac{3\pi}{2} - x) = -\cos x$

$4\sin x(-\cos x) = 1 \rightarrow -2(2\sin x \cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi - \frac{\pi}{6}$ (2) $\quad$ $2x = 2k\pi + \frac{7\pi}{6}$ (1) $\rightarrow x = k\pi - \frac{\pi}{12}$ (2) $\quad$ $k\pi + \frac{7\pi}{12}$ (1)

$\xrightarrow{0 \le x < 2\pi}$ $k = 0, 1 \rightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12}$ (1)

$k = 1, 2 \rightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12}$ (2)

$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$

$1+\cos 2x = 2\cos^2 x$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2 \alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} \cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha \cos^2 8\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} (\underbrace{\sin\alpha \cos\alpha}_{\frac{1}{2}\sin 2\alpha}) \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha \cos 2\alpha}_{\frac{1}{2}\sin 4\alpha}) \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha \cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \cup\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \cup\ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{MAn\,A} k=4 \rightarrow n = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAn\,A} k=2 \rightarrow n = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAn\,A} k=3 \rightarrow n = \frac{7\pi}{9}$$

$$\rightarrow \boxed{MAn\,A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π یا 2π چون طبق $\sin\alpha$ را صفر در نظر گرفته‌ایم!