تکالیف

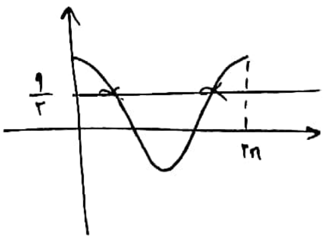
۱-

$8\cos x - \tan^2 x = 1$

$8\cos x = \underbrace{1+\tan^2 x}_{\frac{1}{\cos^2 x}} \longrightarrow 8\cos x = \frac{1}{\cos^2 x} \longrightarrow 8\cos^3 x = 1$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \longrightarrow \begin{cases} x = 2k\pi + \frac{\pi}{3} \\ x = 2k\pi - \frac{\pi}{3} \end{cases}$



۲ جواب دارد. $\longleftarrow$

۲-

$5\sin^2 x + 2\cos 3x = -2$

$2\cos 3x = -2 - 5\sin^2 x \longrightarrow 5 \geq -5\sin^2 x \geq -5$

$-2 \geq -2-5\sin^2 x \geq -7$

$-2 \leq 2\cos 3x \leq 2$

← دو طرف تساوی فقط به ازای ۲- برابر خواهد بود.

$2\cos 3x = -2 \longrightarrow \cos 3x = -1 \longrightarrow 3x = 2k\pi - \pi \longrightarrow x = \frac{2k\pi}{3} - \frac{\pi}{3}$

$-2 - 5\sin^2 x = -2 \longrightarrow \sin^2 x = 0 \longrightarrow x = k\pi$

تعداد جواب ها در بازه $[-\pi, \pi]$ $= \left\{-\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi\right\} \cap \left\{-\pi, 0, \pi\right\} = \left\{-\pi, \pi\right\}$ ۲ جواب دارد.

۳- مجموع جواب ها در بازه $[0, 2\pi]$

$2\sin x \cos 2x + \sin x = 1$

$\sin x\left(\underbrace{2\cos 2x}_{\cos^2 x - \sin^2 x} + 1\right) = \sin x\left(\underbrace{2\cos^2 x}_{1-\sin^2 x} - 2\sin^2 x + 1\right) = \sin x\left(2 - 2\sin^2 x - 2\sin^2 x + 1\right)$

$= \sin x\left(3 - 4\sin^2 x\right) = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1 \longrightarrow \sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2}$

$x = \frac{2k\pi}{3} + \frac{\pi}{6}$

جواب $\rightarrow \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6} \longrightarrow \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{5\pi}{2}$ جواب

نکته بالا

ـ ۴

$$\frac{1}{\sin(\frac{4x+\pi}{2})} + \frac{1}{\cos(\frac{8x+\pi}{2})} = 0 \longrightarrow \frac{\cos(\frac{8x+\pi}{2})+\sin(\frac{4x+\pi}{2})}{\sin(\frac{4x+\pi}{2})\cos(\frac{8x+\pi}{2})}=0$$

$$\sin\left(\frac{4x+\pi}{2}\right)+\cos\left(\frac{8x+\pi}{2}\right)=0$$

$$\sin\left(2x+\frac{\pi}{2}\right)+\cos\left(4x+\frac{\pi}{2}\right)=0 \longrightarrow \cos 2x - \sin 4x = 0$$

$$\cos 2x - 2\sin 2x\cos 2x = 0$$

$$\cos 2x(1-2\sin 2x) = 0$$

$$\cos 2x = 0 \longrightarrow 2x = k\pi+\frac{\pi}{2}$$

$$x = \frac{k\pi}{2}+\frac{\pi}{4}$$

$$1-2\sin 2x = 0 \longrightarrow \sin 2x = \frac{1}{2} \longrightarrow \begin{cases} 2x = 2k\pi+\frac{\pi}{6} \\ 2x = 2k\pi+\frac{5\pi}{6} \end{cases} \longrightarrow \begin{cases} x = k\pi+\frac{\pi}{12} \\ x = k\pi+\frac{5\pi}{12} \end{cases}$$

جوابهای قابل قبول که مخرج را صفر نمی‌کنند $= \frac{\pi}{12}\ ,\ \frac{5\pi}{12}$

$$\alpha = \frac{5\pi}{12}-\frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \qquad \tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$$

جواب

ـ ۶ تعداد جواب‌ها در بازه $[0, 2\pi]$

$$\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{3}\right)=1$$

$$\sin\left(x+\frac{\pi}{6}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{3}+x\right)=1$$

$$\sin^2\left(x+\frac{\pi}{6}\right)=1 \longrightarrow \begin{cases} \sin(x+\frac{\pi}{6})=1 \longrightarrow x=\frac{\pi}{3} \\ \sin(x+\frac{\pi}{6})=-1 \longrightarrow x=\frac{4\pi}{3} \end{cases}$$

۲ جواب دارد.

ـ ۵

$$m(\cos x-\sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \qquad \cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$$

$$\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}} \longrightarrow \cos\left(2x+\frac{\pi}{2}\right) = 2\cos^2\left(x+\frac{\pi}{4}\right)-1 = \frac{2}{3}-1 = \frac{-1}{3}$$

$$-\sin 2x = \frac{-1}{3}$$

$$m(\cos x-\sin x) - \cancel{3}\sqrt{6}\left(\frac{1}{\cancel{3}}\right) = \sqrt{6}$$

$$m\cos x - m\sin x = 2\sqrt{6}$$

$$m^2 \times (1-\underbrace{2\sin x\cos x}_{\sin 2x}) = \frac{24}{\frac{2}{3}} \longrightarrow m=\pm 6$$

جواب

$$\sin 2x = \frac{1}{3}$$

نکات

۸ -

$$4\sin x \sin\left(\frac{3\pi}{2}-x\right)=1$$

$$-4\sin x\cos x=1$$

$$-2(\underbrace{2\sin x\cos x}_{\sin 2x})=1 \longrightarrow \sin 2x=\frac{-1}{2} \longrightarrow 2x=2k\pi+\frac{7\pi}{6}$$

$$x=k\pi+\frac{7\pi}{12}$$

$$2x=2k\pi-\frac{\pi}{6}$$

$$x=k\pi-\frac{\pi}{12}$$

جوابها در بازه $[0,2\pi]$ $=\left\{\frac{7\pi}{12},\frac{19\pi}{12},\frac{11\pi}{12},\frac{23\pi}{12}\right\}$

مجموع جوابها $=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=5\pi$ جواب

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۱۰ -

$$\tan(3x)\tan(x)=1$$

$$\frac{\sin 3x}{\cos 3x}\times\frac{\sin x}{\cos x}=1 \longrightarrow \frac{\sin 3x\sin x}{\cos 3x\cos x}=1$$

$$\sin 3x\sin x=\cos 3x\cos x \longrightarrow \cos 3x\cos x-\sin 3x\sin x=0$$

$$\cos 4x=0 \longrightarrow 4x=k\pi+\frac{\pi}{2}$$

$x=\frac{k\pi}{4}+\frac{\pi}{8}$ ⟵ ۴ جواب دارد

مجموع جوابها در بازه $[\pi,2\pi]$:

مجموع جواب ها = میانگین × تعداد $=\frac{3\pi}{2}\times 4=6\pi$

جواب