۱۷,۷۵

نکات بات

۱-

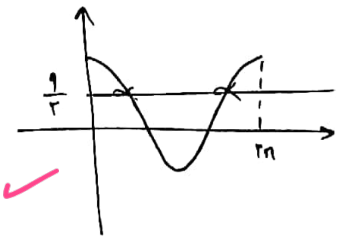
$8\cos x - \tan^2 x = 1$

$8\cos x = \underbrace{1+\tan^2 x}_{\frac{1}{\cos^2 x}} \longrightarrow 8\cos x = \frac{1}{\cos^2 x} \longrightarrow 8\cos^3 x = 1$

$\cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2} \longrightarrow \begin{cases} x = 2k\pi + \frac{\pi}{3} \\ x = 2k\pi - \frac{\pi}{3} \end{cases}$

۲



۲ جواب دارد. ←

۲-

$5\sin^2 x + 2\cos 3x = -2$

$2\cos 3x = -2 - 5\sin^2 x \longrightarrow 5 \ge -5\sin^2 x \ge -5$

$-2 \le 2\cos 3x \le 2$ $\qquad$ $-2 \ge -2-5\sin^2 x \ge -7$

← دو طرف تساوی فقط به ازای ۲- برقرار خواهد بود.

۲

$2\cos 3x = -2 \longrightarrow \cos 3x = -1 \longrightarrow 3x = 2k\pi - \pi \longrightarrow x = \frac{2k\pi}{3} - \frac{\pi}{3}$

$-2 - 5\sin^2 x = -2 \longrightarrow \sin^2 x = 0 \longrightarrow x = k\pi$

تعداد جواب ها در بازه‌ی $[-\pi, \pi]$ $= \{-\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi\} \cap \{-\pi, 0, \pi\} = \{-\pi, \pi\}$ ۲ جواب دارد.

۳- مجموع جواب ها در بازه‌ی $[0, 2\pi]$

$2\sin x \cos 2x + \sin x = 1$

$\sin x(\underbrace{2\cos 2x}_{\cos^2 x - \sin^2 x} + 1) = \sin x(\underbrace{2\cos^2 x}_{1-\sin^2 x} - 2\sin^2 x + 1) = \sin x(2 - 2\sin^2 x - 2\sin^2 x + 1)$

۲

$= \sin x(3 - 4\sin^2 x) = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1 \longrightarrow \sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2}$

$x = \frac{2k\pi}{3} + \frac{\pi}{6}$

جواب ها $\longrightarrow \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6} \longrightarrow \frac{\pi}{6} + \frac{5\pi}{6} + \cancel{\frac{9\pi}{6}} = \frac{5\pi}{2}$

جواب

نکته بالا

-۴

$$\frac{1}{\sin(\frac{4x+\pi}{2})} + \frac{1}{\cos(\frac{8x+\pi}{2})} = 0 \rightarrow \frac{\cos(\frac{8x+\pi}{2})+\sin(\frac{4x+\pi}{2})}{\sin(\frac{4x+\pi}{2})\cos(\frac{8x+\pi}{2})} = 0$$

$$\sin\left(\frac{4x+\pi}{2}\right) + \cos\left(\frac{8x+\pi}{2}\right) = 0$$

۲

$$\sin\left(2x+\frac{\pi}{2}\right) + \cos\left(4x+\frac{\pi}{2}\right) = 0 \rightarrow \cos 2x - \sin 4x = 0$$

$$\cos 2x - 2\sin 2x \cos 2x = 0$$

$$\cos 2x(1-2\sin 2x) = 0$$

$$\cos 2x = 0 \rightarrow 2x = k\pi + \frac{\pi}{2}$$

$$x = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$1-2\sin 2x = 0 \rightarrow \sin 2x = \frac{1}{2} \rightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{6} \\ 2x = 2k\pi + \frac{5\pi}{6} \end{cases} \rightarrow \begin{cases} x = k\pi + \frac{\pi}{12} \\ x = k\pi + \frac{5\pi}{12} \end{cases}$$

جوابهای قابل قبول که مخرج را صفر نمی‌کنند $= \frac{\pi}{12} , \frac{5\pi}{12}$

$$\alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3} \qquad \tan 2\alpha = \tan\frac{2\pi}{3} = -\sqrt{3}$$ جواب

۶- تعداد جواب ها در بازه $[0, 2\pi]$ $\qquad \sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3}) = 1$

$$\sin\left(x+\frac{\pi}{6}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{3}+x\right) = 1$$

۲

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \rightarrow \begin{cases} \sin(x+\frac{\pi}{6}) = 1 \rightarrow x = \frac{\pi}{3} \\ \sin(x+\frac{\pi}{6}) = -1 \rightarrow x = \frac{4\pi}{3} \end{cases}$$

۲ جواب دارد.

۵- $m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \qquad \cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

۱/۷۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \rightarrow \cos\left(2x+\frac{\pi}{2}\right) = 2\cos^2\left(x+\frac{\pi}{4}\right) - 1 = \frac{2}{3} - 1 = \frac{-1}{3}$$

$$-\sin 2x = \frac{-1}{3}$$

$$m(\cos x - \sin x) - \cancel{3}\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\sin 2x = \frac{1}{3}$$

$$m\cos x - m\sin x = 2\sqrt{6}$$

$$m^2 \times (1 - \underbrace{2\sin x\cos x}_{\sin 2x}) = \frac{24}{\frac{2}{3}} \rightarrow m = \pm 6$$

جواب

نکات :

-۸

$4\sin x \sin\left(\frac{3\pi}{2}-x\right)=1$

$-4\sin x \cos x = 1$

$-2(\underbrace{2\sin x \cos x}_{\sin 2x}) = 1 \longrightarrow \sin 2x = \frac{-1}{2} \longrightarrow 2x = 2k\pi + \frac{7\pi}{6}$

$x = k\pi + \frac{7\pi}{12}$

$\longrightarrow 2x = 2k\pi - \frac{\pi}{6}$

$x = k\pi - \frac{\pi}{12}$

جوابها در بازه $[0, 2\pi]$ $= \left\{ \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}, \frac{23\pi}{12} \right\}$

مجموع جوابها $= \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$ جواب

-۱۰

$\tan(3x)\tan(x) = 1$

$\frac{\sin 3x}{\cos 3x} \times \frac{\sin x}{\cos x} = 1 \longrightarrow \frac{\sin 3x \sin x}{\cos 3x \cos x} = 1$

$\sin 3x \sin x = \cos 3x \cos x \longrightarrow \cos 3x \cos x - \sin 3x \sin x = 0$

$\cos 4x = 0 \longrightarrow 4x = k\pi + \frac{\pi}{2}$

۴ جواب دارد $\longleftarrow$ $x = \frac{k\pi}{4} + \frac{\pi}{8}$

مجموع جواب ها $=$ تعداد $\times$ میانگین $= \frac{3\pi}{2} \times 4 = 6\pi$ جواب

مجموع جوابها در بازه $[\pi, 2\pi]$:

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3} m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3} m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m = 6}$$

۹

$$1 + \cos 2\star = 2\cos^2\star$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \longrightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} \star$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2} \times \frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}} \sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا}\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا}\ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\text{MAX } A} k = 3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAX } A} k = 4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\text{MAX } A} k = 2 \longrightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\text{MAX } A} k = 3 \longrightarrow x = \frac{7\pi}{9}$$

$$\longrightarrow \boxed{\text{MAX } A = \frac{8\pi}{9}}$$

$\star$ A نمی‌تواند برابر خود π باشد چون طبق $\star$ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!