نام و نام خانوادگی ...لیلی صباحی............  پاسخنامه تشریحی تکلیف شماره ..۱۷.. کلاس دوازدهم...

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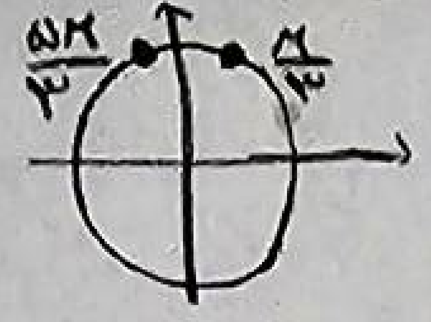
$8\cos x - \tan^2 x = 1 \Rightarrow 8\cos x - \frac{1-\cos^2 x}{\cos^2 x} = 1 \Rightarrow 8\cos^3 x - 1 + \cancel{\cos^2 x} = \cancel{\cos^2 x} \Rightarrow$

$\cos x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$




$\Rightarrow$ $\boxed{\text{۲ جواب}}$

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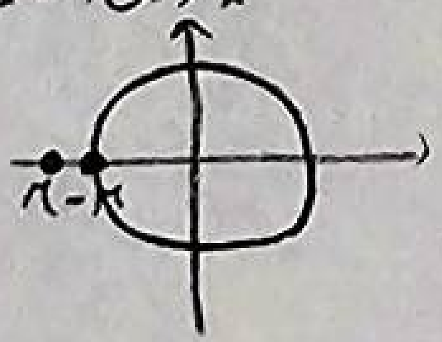
$5\underbrace{\sin^2 x}_{1-\cos^2 x} + 2\underbrace{\cos 3x}_{4\cos^3 x - 3\cos x} = -2 \Rightarrow 8\cos^3 x - 6\cos x - 5\cos^2 x + 7 = 0 \Rightarrow (\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\Delta < 0} = 0$

$\Rightarrow \cos x = -1$




$\Rightarrow$ $\boxed{\text{۲ جواب}}$

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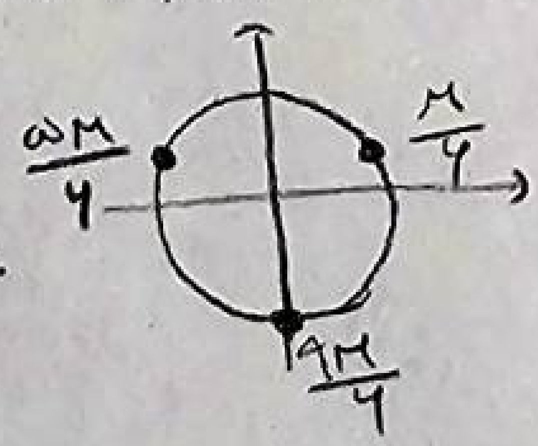
$2\sin x \underbrace{\cos 2x}_{1-2\sin^2 x} + \sin x = 1 \Rightarrow 2\sin x - 4\sin^3 x + \sin x = 1 \Rightarrow 3\sin x - 4\sin^3 x = 1 \Rightarrow \sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6}$




$\frac{5\pi}{6} + \frac{\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$\frac{1}{\underbrace{\sin(\frac{\pi}{2}+2x)}_{\cos 2x}} + \frac{1}{\underbrace{\cos(\frac{\pi}{2}+4x)}_{-\sin 4x}} = 0 \Rightarrow \frac{1}{\cos 2x} = \frac{1}{\sin 4x} \Rightarrow \sin 4x = \cos 2x \Rightarrow \sin 4x = 2\sin 2x\cos 2x$

$\Rightarrow 2\sin 2x\cos 2x = \cos 2x \Rightarrow \cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + k\pi \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4} \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}$

$\sin 2x = \frac{1}{2} \Rightarrow x = \frac{\pi}{12} + k\pi \Rightarrow \frac{\pi}{12}$

$x = \frac{5\pi}{12} + k\pi \Rightarrow \frac{5\pi}{12}$

$\frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3} = \alpha$

$2\alpha = \frac{2\pi}{3} \Rightarrow \tan\frac{2\pi}{3} = -\tan\frac{\pi}{3} = \boxed{-\sqrt{3}}$

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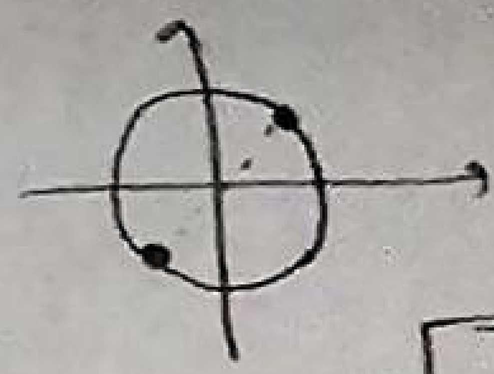
$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x$ و $\cos(x+\frac{\pi}{4}) = \frac{\sqrt{3}}{3} = \frac{\sqrt{2}}{2}(\cos x - \sin x) \Rightarrow \cos x - \sin x$

$= \frac{2}{\sqrt{6}} \Rightarrow \sin 2x = 2\sin x\cos x \Rightarrow (\cos x - \sin x)^2 = (\frac{2}{\sqrt{6}})^2 \Rightarrow 1 - \sin 2x = \frac{2}{3} \Rightarrow \sin 2x = \frac{1}{3}$

$\Rightarrow \sqrt{6} = m(\frac{2}{\sqrt{6}}) - 3\sqrt{6}(\frac{1}{3}) \Rightarrow 2\sqrt{6} = \frac{2m}{\sqrt{6}} \Rightarrow \boxed{m = 6}$

$\sin(x+\frac{\pi}{6})\underbrace{\cos(x-\frac{\pi}{3})}_{\sin(\frac{\pi}{6}+x-\frac{\pi}{3})}=1 \Rightarrow \sin(x+\frac{\pi}{6})\cdot\sin(x+\frac{\pi}{6})=1 \Rightarrow \sin^2(x+\frac{\pi}{6})=1 \Rightarrow x+\frac{\pi}{6}=k\pi+\frac{\pi}{2}$ ۶

$\Rightarrow x=k\pi+\frac{\pi}{3}$



$\boxed{\text{۲ جواب}}$

$\sin x+\sqrt{3}\cos x=\sqrt{2} \Rightarrow \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2} \Rightarrow \underbrace{\cos x=\frac{1}{2},\ \sin x=\frac{\sqrt{3}}{2}}_{x=\frac{\pi}{3}}$

$\Rightarrow \cos\frac{\pi}{3}\sin x+\sin x\cos\frac{\pi}{3}=\frac{\sqrt{2}}{2} \Rightarrow \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$ ۷

$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4} \Rightarrow x=2k\pi-\frac{\pi}{12} \Rightarrow x=-\frac{\pi}{12},\ \frac{23\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\frac{3\pi}{4} \Rightarrow x=2k\pi+\frac{5\pi}{12} \Rightarrow x=\frac{5\pi}{12}$

$\Big\}\ -\frac{\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}=\boxed{\frac{9\pi}{4}}$

$4\sin x\sin(\frac{3\pi}{2}-x)=1 \Rightarrow -2(2\sin x\cos x)=1 \Rightarrow -2\sin 2x=1 \Rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi+\frac{7\pi}{6} \Rightarrow x=\frac{7\pi}{12}+k\pi \Rightarrow x=\frac{7\pi}{12},\ \frac{19\pi}{12}$

$2x=2k\pi+\frac{11\pi}{6} \Rightarrow x=\frac{11\pi}{12}+k\pi \Rightarrow x=\frac{11\pi}{12},\ \frac{23\pi}{12}$ ۸

$\Big\}\ \frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$

$|(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)|=\frac{1}{8}$, with $1+\cos 2\alpha=2\cos^2\alpha$, $1+\cos 4\alpha=2\cos^2 2\alpha$, $1+\cos 8\alpha=2\cos^2 4\alpha$

$\Rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{8} \Rightarrow |\cos\alpha\cos 2\alpha\cos 4\alpha|=\frac{1}{8}$

$\Rightarrow |4(2\sin\alpha\cos\alpha)(\cos 2\alpha\cos 4\alpha)|=|\sin\alpha| \Rightarrow 2|2\sin 2\alpha\cos 2\alpha\cos 4\alpha|=|\sin\alpha|$ ۹

$\Rightarrow |2\sin 4\alpha\cos 4\alpha|=|\sin\alpha| \Rightarrow |\sin 8\alpha|=|\sin\alpha| \Rightarrow \sin 8\alpha=\sin\alpha \Rightarrow 8\alpha=2k\pi+\pi \Rightarrow \alpha=\frac{2k\pi}{7}$ ... $\frac{2\pi}{7}$ (1)

$8\alpha=2k\pi+\pi-\alpha \Rightarrow \alpha=\frac{2k\pi+\pi}{9}$ ... $\pi$ (2)

$\sin 8\alpha=-\sin\alpha=\sin(-\alpha) \Rightarrow \alpha=\frac{2k\pi}{9},\ \alpha=\frac{2k\pi+\pi}{7}$ (3) $\pi=$ (1), (2), (3), (4) $\Rightarrow \boxed{\frac{8\pi}{9}}$

(4) $\frac{8\pi}{9}$

$\tan 3x\tan x=1 \Rightarrow \tan 3x=\frac{1}{\tan x} \Rightarrow \tan 3x=\cot x \Rightarrow \tan 3x=\tan(\frac{\pi}{2}-x)$

$\Rightarrow 3x=k\pi+\frac{\pi}{2}-x \Rightarrow x=\frac{2k\pi+\pi}{8} \Rightarrow x=\frac{9\pi}{8},\ \frac{11\pi}{8},\ \frac{13\pi}{8},\ \frac{15\pi}{8} \Rightarrow$ ۱۰

$\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=\boxed{6\pi}$