۱۹ آفرین

بنام خدا

«مطهره کمیلی»

«سوالات طیف سری ۱۷»

سوال ۱ ←

$8\cos x - \tan^2 x = 1 \longrightarrow 8\cos x - \underbrace{(\tan^2+1)}_{\frac{1}{\cos^2 x}} = 0 \quad \times \cos^2 x \longrightarrow 8\cos^3 x - 1 = 0$

$\cos^3 x = \frac{1}{8} \longrightarrow \boxed{\cos x = \frac{1}{2}}$

$\cos x = \cos\frac{\pi}{3} \longrightarrow x = 2k\pi \pm \frac{\pi}{3}$

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سوال ۲ ←

$5\sin^2(x) + 2\cos(3x) = 2 \longrightarrow 5 - 5\cos^2 x + (4\cos^3 x - 3\cos x)\times 2 = 2 \longrightarrow 8\cos^3 x - 5\cos^2 x - 6\cos x + 3 = 0$

$5\sin^2 x \to 5(1-\cos^2 x)$ ، $\cos 3x \to 4\cos^3 x - 3\cos x$

فرد = جمع ضرایب زوج ← جمع ضرایب = ۰ ← یکی از ریشه‌ها (-1) ← $(\cos x + 1) \div$ ←

$\boxed{8\cos^2 x - 13\cos x + 3}$

$\Delta < 0 \longrightarrow$ جواب ندارد

$\cos x = \cos \pi \to x = 2k\pi \pm \pi \longrightarrow \boxed{x = \pi}$ و $\boxed{x = -\pi}$

۲

سوال ۳ ←

$2\sin x \times \cos(2x) + \sin x = 1 \longrightarrow 2\sin x(-2\sin^2+1) + \sin x - 1 = 0$

$\cos(2x) \to (-2\sin^2 x + 1)$

$-4\sin^3 x + 2\sin x + \sin x - 1 = 0$

$-4t^3 + 3t - 1 = 0 \xrightarrow[\div]{(t+1)} (-4t^2 + 4t - 1) \to -(2t-1)^2$

ریشه ← $t = \frac{1}{2}$ ، $t = -1$ ← $t = \sin x$ ←

$\sin x = \frac{1}{2} \to \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = -1 \to \frac{3\pi}{2}$

$\xleftarrow{+} \quad \frac{\pi}{6} + \frac{5\pi}{6} + \frac{3}{2}\pi = \boxed{\frac{5\pi}{2}}$

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سوال ۴ ←

$\frac{1}{\text{☁}} + \frac{1}{\triangle} = 0 \longrightarrow \text{☁} = -\triangle \longrightarrow \cos\left(\frac{\pi + 8x}{2}\right) = -\sin\left(+\frac{\pi + 4x}{2}\right)$

تبدیل cos به sin $\longrightarrow \sin(4x) = \sin\left(\frac{\pi}{2} + 2x\right)$

$\to 4x = \frac{\pi}{2} + 2x + 2k\pi \longrightarrow x = k\pi + \frac{\pi}{4} \to \frac{\pi}{4}$

$\to 4x = 2k\pi + \pi - \frac{\pi}{2} - 2x \longrightarrow x = \frac{k\pi}{3} + \frac{\pi}{12} \to \frac{\pi}{12}, \frac{\pi}{12} + \frac{2\pi}{3}$

$\frac{2\pi}{3} + \frac{\pi}{12}$

$\sin\left(\frac{\pi + 4x}{2}\right) \neq 0$ و $\cos\left(\frac{\pi + 8x}{2}\right) \neq 0 \longrightarrow x \neq \frac{\pi}{4}$ ، $x \neq \frac{2\pi}{3} + \frac{\pi}{12}$

$\longrightarrow$ جواب‌های ما $= \frac{\pi}{12}, \frac{\pi}{12} + \frac{2\pi}{3} \longrightarrow$ تفاضل (120)

$\boxed{\tan 2\alpha = \tan 120 = -\sqrt{3}}$

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سوال ۵ ←

$\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}} \longrightarrow -\frac{1}{\sqrt{2}}(\sin x - \cos x) = \frac{1}{\sqrt{3}} \to (\sin x - \cos x)^2 = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 \to \underbrace{\sin^2 x + \cos^2 x}_{1} - 2\sin x\cos x = \frac{2}{3}$

$\sin\left(\frac{\pi}{4} - x\right) \to -\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$

$-2\sin x\cos x = -\frac{1}{3}$

$\boxed{\sin 2x = \frac{1}{3}}$

$m\,(\underbrace{\cos x - \sin x}_{\pm\frac{\sqrt{2}}{\sqrt{3}}}) - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \longrightarrow m = \frac{\pm 2\sqrt{6}\times\sqrt{3}}{\sqrt{2}} = \boxed{m = \pm 6}$

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سوال ۶ ←

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1$ $\xrightarrow{\text{زاویه‌ی متمم}}$ $x+\frac{\pi}{6}-x+\frac{\pi}{3}=\frac{\pi}{2}$

$\sin^2(x+\frac{\pi}{6})=1 \rightarrow \sin(x+\frac{\pi}{6})=\pm 1$

$\hookrightarrow \cos(-x+\frac{\pi}{3})$

$x+\frac{\pi}{6}=\frac{\pi}{2} \rightarrow x=\frac{\pi}{3}$

$x+\frac{\pi}{6}=\frac{3\pi}{2} \rightarrow x=\frac{4\pi}{3}$

$x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2} \rightarrow x=2k\pi+\frac{\pi}{3}$

$x+\frac{\pi}{6}=2k\pi+\pi-\frac{\pi}{2} \rightarrow x=2k\pi+\frac{\pi}{2}-\frac{\pi}{6} \rightarrow x=2k\pi-\frac{2\pi}{3}$

۲ جواب دارد

سوال ۷ ←

$\frac{\sin x+\sqrt{3}\cos x}{2}=\frac{\sqrt{2}}{2} \rightarrow \sin(x+\frac{\pi}{3})=\sin(\frac{\pi}{4}) \rightarrow x+\frac{\pi}{3}=\frac{\pi}{4} \rightarrow x=-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \rightarrow x=\frac{5\pi}{12}$

مجموع $=-\frac{\pi}{12}+\frac{5\pi}{12}+\frac{11\pi}{12}=\frac{15\pi}{12}=\frac{5\pi}{4}$

سوال ۸ ←

$4\sin x\sin(\frac{3\pi}{2}-x)=1 \rightarrow -2\sin(2x)=1 \rightarrow \sin(2x)=-\frac{1}{2}$

$\hookrightarrow -\cos(-x)=-\cos(x)$

$\sin^2 2x \rightarrow \frac{1}{2}\sin^2 4x \rightarrow \frac{1}{4}\sin^2 8x$

$\frac{2\sin^2 x\times 2\cos^2 x\times 2\cos^2 2x\times 2\cos^2 4x}{2\sin^2 x}=\frac{1}{8} \rightarrow \frac{\frac{1}{8}\sin^2 8x}{\sin^2 x}=\frac{1}{8} \rightarrow \sin x=\sin 8x$

سوال ۹ ←

$\sin(x)=\sin 8x$

$-x+2k\pi+\pi=8x \rightarrow 9x=2k\pi+\pi \rightarrow x=\frac{2k\pi+\pi}{9}$ $\xrightarrow{\text{min}}$ $\frac{\pi}{9}$ $\xrightarrow{\text{کوچکتر } \pi}$ جواب مورد نظر

$x+2k\pi+\pi=8x \rightarrow 7x=2k\pi+\pi \rightarrow x=\frac{2k\pi+\pi}{7}$ $\xrightarrow{\text{min}}$ $\frac{\pi}{7}$ $\xrightarrow{\text{کوچکتر } \pi}$ $\frac{5\pi}{7}$

سوال ۱۰ ←

$\tan(x)\tan(3x)=1 \rightarrow \tan x=\frac{1}{\tan(3x)} \rightarrow \tan x=\cot(3x)$

$\hookrightarrow \tan(\frac{\pi}{2}-3x)$

$x=\frac{\pi}{2}-3x \rightarrow 4x=\frac{\pi}{2} \rightarrow x=\frac{\pi}{8}$

$x\neq 2k\pi+\pi-\frac{\pi}{2}+3x \rightarrow -2x=2k\pi+\frac{\pi}{2} \rightarrow x=-k\pi-\frac{\pi}{4}$ $\rightarrow k\in \mathbb{Z}$ طبق دامنه فقط $k=0$ قابل قبول می‌باشد

$\rightarrow$ جواب کل $=2k\pi+\frac{\pi}{8}$

(۸)

$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$

$4\sin x(-\cos x)=1 \rightarrow -2(2\sin x\cos x)=1 \rightarrow \sin 2x=-\frac{1}{2}$

$2x=2k\pi-\frac{\pi}{6}$ (2) $\quad 2k\pi+\frac{7\pi}{6} \rightarrow x=k\pi-\frac{\pi}{12}$ (2) $\quad k\pi+\frac{7\pi}{12}$

$0\le x\le 2\pi$ (1)

$k=0,1 \rightarrow x=\frac{7\pi}{12}, \frac{19\pi}{12}$

$k=1,2 \rightarrow x=\frac{11\pi}{12}, \frac{23\pi}{12}$

$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$

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$\tan 3x=\frac{1}{\tan x}=\cot x \rightarrow \tan 3x=\tan\left(\frac{\pi}{2}-x\right)$

$3x=k\pi+\frac{\pi}{2}-x \rightarrow 4x=k\pi+\frac{\pi}{2} \rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$

k	۴	۵	۶	۷
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$S=\left(\frac{9+11+13+15}{8}\right)\pi=\frac{48\pi}{8}=\boxed{6\pi}$

(۹)

$1+\cos 2\theta = 2\cos^2\theta$

$\int 1+\cos 2\alpha = 2\cos^2\alpha$

$\left| \begin{array}{l} 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{array} \right.$? $\rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$

$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha} *$

$(\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha})\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha\cos 2\alpha}_{\frac{1}{2}\sin 4\alpha})\cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha\cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$

$\sin 8\alpha = \pm\sin\alpha \rightarrow \left| \begin{array}{l} 8\alpha = 2k\pi + \alpha \ \cup\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \cup\ 2k\pi + \pi + \alpha \end{array} \right.$

$\alpha = \frac{2k\pi}{7} \xrightarrow{MAn\,A} k=2 \rightarrow n = \frac{4\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{MAn\,A} k=4 \rightarrow n = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAn\,A} k=2 \rightarrow n = \frac{5\pi}{7}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAn\,A} k=4 \rightarrow n = \frac{7\pi}{9}$

$\rightarrow \boxed{MAn\,A = \frac{8\pi}{9}}$

* A نمی‌تواند برابر خود π باشد چون طبق $\sin\alpha$ را مضارب صفر در نظر گرفته‌ایم!