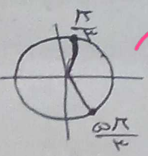
تکلیف ۱۷ — [illegible] — ۱۵

(۱) ۲

$8\cos x = \tan^2 x + 1 \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1$

$2\cos x = 1 \Rightarrow \cos x = \frac{1}{2} \qquad x = 2k\pi \pm \frac{\pi}{3}$



دارای ۲ جواب است.

(۲) ۰

$\cos 3x = 2\cos^2 \frac{3x}{2} - 1$ (فرمول طلایی)

$5\sin^2 x + 2(2\cos^2\frac{3x}{2} - 1) = -2 \rightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} - \cancel{2} = \cancel{-2}$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$5\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi$ یا $-\pi$ یا 0

$4\cos^2\frac{3x}{2} = 0 \rightarrow \cos\frac{3x}{2} = 0 \rightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi + \pi}{3}$

$\xrightarrow{-\pi \le x \le \pi} x = -\pi$ یا $-\frac{\pi}{3}$ یا $\frac{\pi}{3}$ یا π

اشتراک مجموعه جواب $\{\pi, -\pi\}$ است

(۳) ۰

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \rightarrow \underbrace{3\sin x - 4\sin^3 x}_{\sin 3x} = 1$

$\sin 3x = 1 \rightarrow 3x = 2k\pi + \frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$

$k=0 \rightarrow x = \frac{\pi}{6}, \quad k=1 \rightarrow x = \frac{5\pi}{6}, \quad k=2 \rightarrow x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$\frac{1}{\sin(\frac{\pi}{2} + 4x)} + \frac{1}{\cos(\frac{\pi}{2} + 4x)} = 0 \Rightarrow \frac{1}{\cos 4x} - \frac{1}{\sin 4x} = 0 \Rightarrow \frac{1}{\sin 4x} = \frac{1}{\cos 4x}$

$\sin 4x = \cos 4x \Rightarrow \sin 4x = \cos 2x \Rightarrow 2\sin 2x \cos 2x = \cos 2x$

$\cos 2x \ne 0 \Rightarrow \sin 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi + \frac{\pi}{6}$ یا $2x = 2k\pi + \frac{5\pi}{6}$

$x = k\pi + \frac{\pi}{12} \qquad x = k\pi + \frac{5\pi}{12} \qquad \alpha = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$x = \frac{\pi}{12} \qquad x = \frac{5\pi}{12}$

(۵) ۱٫۵

$\boxed{\tan\frac{2\pi}{3} = -\sqrt{3}}$

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x \Rightarrow \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow \cos x - \sin x = \frac{2}{\sqrt{6}}$

$m(\frac{2}{\sqrt{6}}) - 2\sqrt{6}\sin 2x = \sqrt{6}$

تشریح سوالات

(۶)

$$\sin(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})=1 \underset{\cos x=\cos(-x)}{\Longrightarrow} \sin(x+\frac{\pi}{4})\cos(-x+\frac{\pi}{4})=1$$

$x+\frac{\pi}{4}+(-x+\frac{\pi}{4})=\frac{\pi}{2}$ زوایه متمم اند $\Longrightarrow \sin^2(x+\frac{\pi}{4})=1$ ① $\sin(x+\frac{\pi}{4})=1$ ② $\sin(x+\frac{\pi}{4})=-1$

$\sin x=1 \quad x+\frac{\pi}{4}=2k\pi+\frac{\pi}{2} \Longrightarrow x=2k\pi+\frac{\pi}{4} \quad x=\frac{\pi}{4}$

$\sin x=-1 \quad x+\frac{\pi}{4}=2k\pi-\frac{\pi}{2} \Longrightarrow x=2k\pi-\frac{3\pi}{4} \quad x=\frac{5\pi}{4}$

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(۷)

$a\sin x+b\cos x \Longrightarrow \frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$, $\tan\alpha=\frac{b}{a}$

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$2\sin(x+\frac{\pi}{3})=\sqrt{2} \Longrightarrow \sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2} \Longrightarrow x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}$

$x=2k\pi-\frac{\pi}{12}$

$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4} \Longrightarrow x=2k\pi+\frac{5\pi}{12}$

$x=\frac{5\pi}{12}$ $\quad x=-\frac{\pi}{12}, \frac{23\pi}{12}$

$-\frac{\pi}{12}+\frac{5\pi}{12}+\frac{23\pi}{12}=\frac{27\pi}{12}$

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$4\sin x\sin(\underbrace{\frac{3\pi}{2}-x}_{-\cos x})=1 \Longrightarrow -4\sin x\cos x=1 \Longrightarrow +2\sin x\cos x=-\frac{1}{2}$

$\sin 2x=-\frac{1}{2}$ $\quad 2x=2k\pi-\frac{\pi}{6}$, $2x=2k\pi+\frac{7\pi}{6}$

$x=k\pi-\frac{\pi}{12}$ $\quad x=k\pi+\frac{7\pi}{12}$

$x=\frac{11\pi}{12}, \frac{23\pi}{12}$ $\quad x=\frac{7\pi}{12}, \frac{19\pi}{12}$

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$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$

(۹)

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8} \Longrightarrow (2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$

$8(\cos\alpha\cos 2\alpha\cos 4\alpha)^2=\frac{1}{8} \Longrightarrow \cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}$

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$\xrightarrow{\alpha\ 8\sin\alpha}$ $8\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}\,\alpha\,8\sin\alpha$

$4\alpha\ \frac{2\sin\alpha\cos\alpha}{\sin 2\alpha}$ $\quad \frac{2\sin 2\alpha\cos 2\alpha}{\sin 4\alpha}$ $\quad \frac{2\sin 4\alpha\cos 4\alpha}{\sin 8\alpha}$

① $\sin 8\alpha=+\sin\alpha$

② $\sin 8\alpha=-\sin\alpha$ $\quad \sin(-\alpha)$

① $8\alpha=2k\pi+\alpha$, $8\alpha=2k\pi+\pi-\alpha$ $\quad \alpha=\frac{2k\pi}{7}$, $\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$

② $8\alpha=2k\pi-\alpha$, $8\alpha=2k\pi+\pi+\alpha$ $\quad \alpha=\frac{2k\pi}{9}$, $\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$

$\max=\frac{2\times 4\pi}{9}+\frac{\pi}{9}=\pi$

(۱۰)

$\tan 3x=\cot x \Longrightarrow \tan 3x=\tan(\frac{\pi}{2}-x) \Longrightarrow 3x=k\pi+\frac{\pi}{2}-x$

$\tan 3x=\frac{1}{\tan x}$ $\quad x=\frac{k\pi}{4}+\frac{\pi}{8}$

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$x=\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \Longrightarrow \frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=\frac{48\pi}{8}=6\pi$

۵

$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m = 6}$$

۹

$$1 + \cos 2* = 2\cos^2 *$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{\text{ضرب}} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha\ *}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{7\pi}{9}$$

$$\longrightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!