فاطمه رستمیان

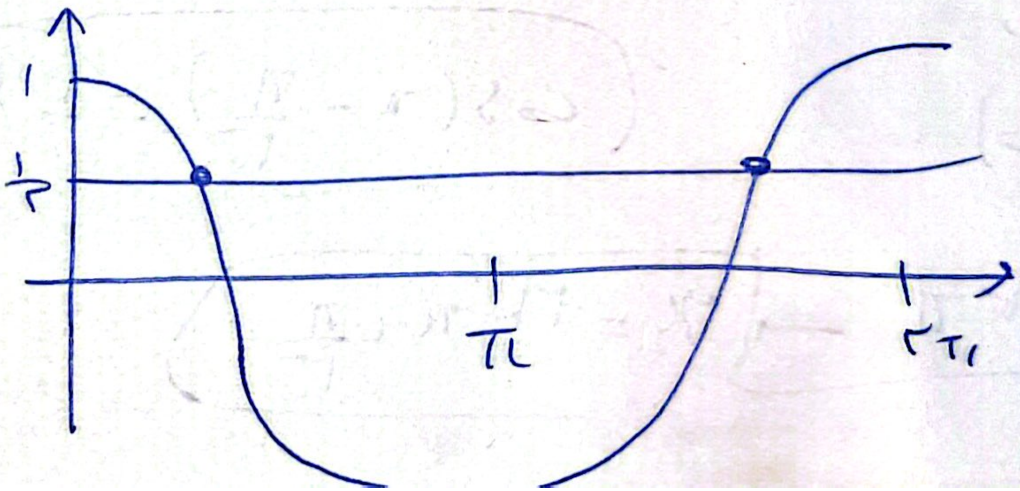
۱۲ اس شنبه

سوال ۱)

$8\cos - \tan^2 = 1 \rightarrow 8\cos = 1 + \tan^2$

$1 + \tan^2 = \frac{1}{\cos^2}$ $\quad 8\cos = \frac{1}{\cos^2} \rightarrow \cos^3 = \frac{1}{8} \rightarrow \boxed{\cos = \frac{1}{2}}$

$[0, 2\pi) \longrightarrow$



سوال ۲

فاطمه سهیل

سوال ۲

$5\sin^2 x + 2\cos 2x = -2$

$2\cos 2x = -2 - 5\sin^2 x \rightarrow 0 \geq -5\sin^2 x \geq -5$

$-2 \geq -2 - 5\sin^2 x \geq -7$

$-2 \leq 2\cos 2x \leq 2$

دو طرف تساوی فقط به ازای ۲- می‌توانند برابر باشند

$2\cos 2x = -2 \qquad \cos 2x = -1 \qquad 2x = 2k\pi - \pi \rightarrow x = \frac{2k\pi}{2} - \frac{\pi}{2}$

$-2 - 5\sin^2 x = -2 \rightarrow \sin^2 x = 0 \rightarrow x = k\pi$

تعداد جواب در بازه $[-\pi, \pi]$

$= \{-\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \pi\} \cap \{-\pi, 0, \pi\} = \{-\pi, \pi\}$

۲ جواب دارد

$(-\pi, \pi]$

۲ جواب دارد

سوال ۵

$2\sin x\cos^2 x + \sin x = 1$

$\sin x(2\cos^2 x + 1) = \sin x(2\cos^2 - 2\sin^2 + 1) = \sin x(2 - 2\sin^2 x - 2\sin^2 x + 1)$

$= \sin x(3 - 4\sin^2 x) = 1 \to 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2}$

$x = \frac{2k\pi}{3} + \frac{\pi}{6}$

۳ جواب $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{5\pi}{2}$

$$\frac{1}{\sin(4x+\frac{\pi}{2})}+\frac{1}{\cos(8x+\frac{\pi}{2})}=0$$

$$\Leftrightarrow \sin(4x+\tfrac{\pi}{2})+\cos(8x+\tfrac{\pi}{2})=$$

$$\sin(4x+\tfrac{\pi}{2})+\cos(8x+\tfrac{\pi}{2})=0 \rightarrow \cos 4x-\sin 8x=0$$

$$\cos 4x-2\sin 4x\cos 4x=0$$

$$\cos 4x(1-2\sin 4x)=0$$

$$\cos 4x=0 \rightarrow 4x=k\pi+\frac{\pi}{2}$$

$$x=\frac{k\pi}{4}+\frac{\pi}{8}$$

$$1-2\sin 4x=0 \rightarrow \sin 4x=\frac{1}{2} \rightarrow \begin{cases}4x=2k\pi+\frac{\pi}{6}\\ 4x=2k\pi+\frac{5\pi}{6}\end{cases} \rightarrow \begin{array}{l}x=\frac{k\pi}{2}+\frac{\pi}{24}\\ x=\frac{k\pi}{2}+\frac{5\pi}{24}\end{array}$$

$$a=\frac{5\pi}{24}-\frac{\pi}{24}=\frac{4\pi}{24}=\frac{\pi}{6}$$

(سوال ۴)

$$\sin\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{2}\right)=1$$

$$\left(x-\frac{\pi}{2}+\frac{\pi}{2}\right)$$

$$\cos\left(x-\frac{\pi}{2}\right)$$

$$\cos\left(x-\frac{\pi}{2}\right)^{2}=1$$

$$\cos\left(x-\frac{\pi}{2}\right)=\pm 1 \rightarrow \begin{matrix} 2k\pi+0 \\ 2k\pi+\pi \end{matrix}$$

$$x-\frac{\pi}{2}=2k\pi \rightarrow \boxed{x=2k\pi+\frac{\pi}{2}}$$

$$x-\frac{\pi}{2}=2k\pi+\pi \rightarrow x=2k\pi+\pi+\frac{\pi}{2} \rightarrow \boxed{x=2k\pi+\frac{3\pi}{2}}$$

$$\frac{\pi}{2} \qquad \frac{3\pi}{2}$$

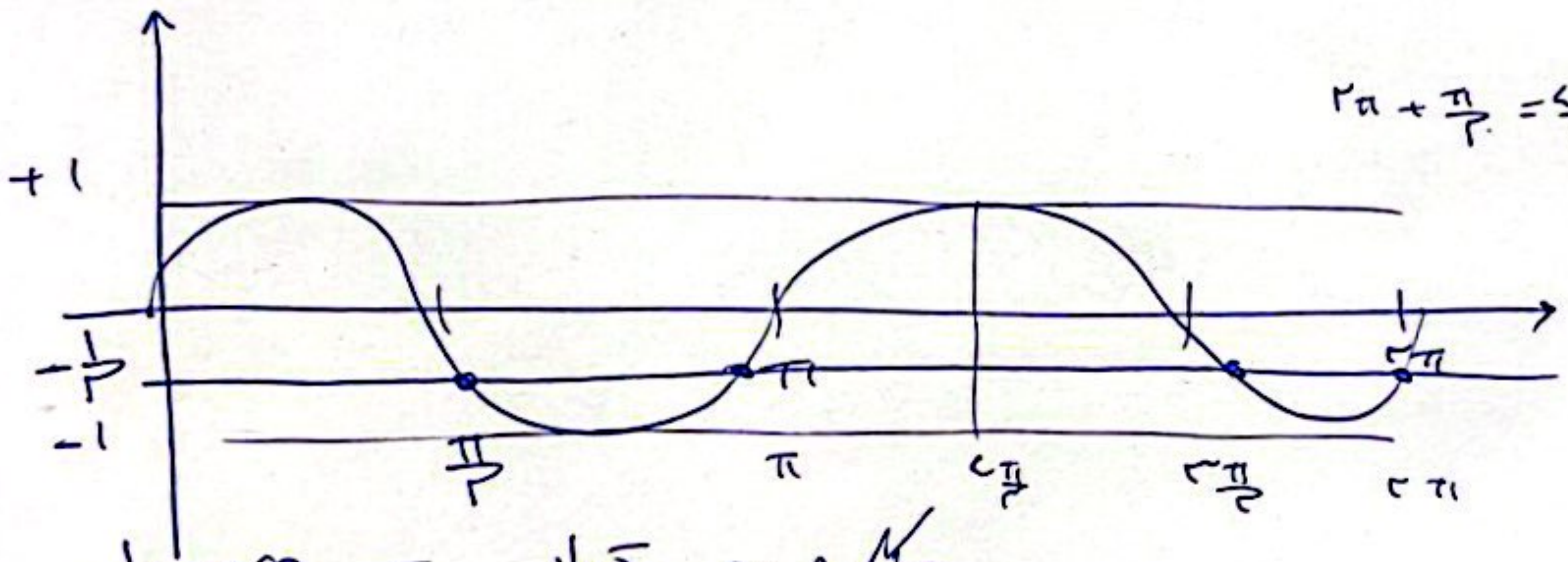
۲ ملب

سوال ۸

$$4\sin x \underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x} = 1$$

$$-4\sin x\cos x = 1 \rightarrow \underbrace{\sin x\cos x}_{\frac{1}{2}\sin 2x} = \frac{-1}{4} \rightarrow \boxed{\sin 2x = -\frac{1}{2}}$$

$$2\pi + \frac{\pi}{2} = \frac{5\pi}{2} \div 2 = \frac{5\pi}{4}$$



مجموع جواب = تعداد × میانگین

$$4 \times \frac{5\pi}{4} = 5\pi$$

Scanned with CamScanner

سوال ۱۰

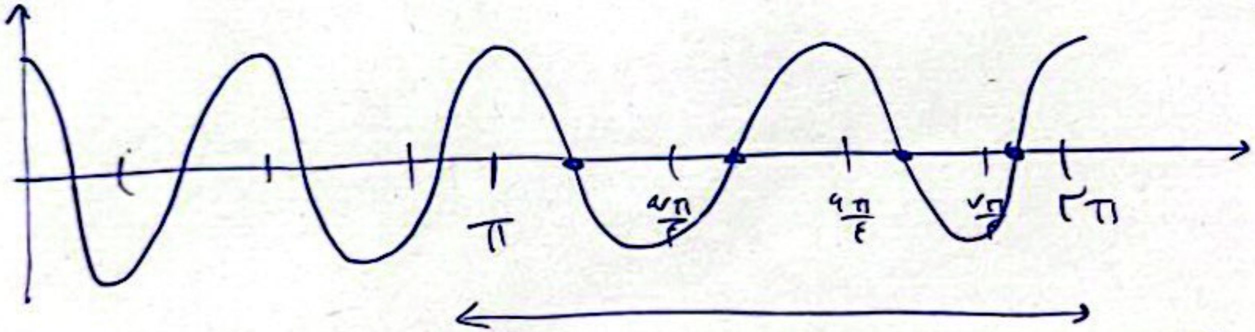
$$\frac{\sin ۳x}{\cos ۳x} \times \frac{\sin x}{\cos x} = ۱ \rightarrow \frac{\sin ۳x \sin x}{\cos ۳x \cos x} = ۱$$

$$\sin ۳x \sin x = \cos ۳x \cos x \rightarrow \cos ۳x \cos x - \sin ۳x \sin x = ۰$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

$$\cos(۴x) = ۰$$

معمول



$$\text{مجموع} = \text{تعداد} \times \text{میانگین} \Rightarrow \frac{۳\pi}{\cancel{۲}} \times ۴ = ۶\pi$$

سوال

$m(\cos x - \sin x) - ٣\sqrt{٦} \sin(٢x) = \sqrt{٦}$

$m(\cos x - \sin x) = ٢\sqrt{٦}$

$m^{٢}(١ - ٢\sin x \cos x) = \dfrac{٢٤}{\frac{٢}{٣}} = m = \pm ٦$

$\cos\left(x + \dfrac{\pi}{٤}\right) = \dfrac{١}{\sqrt{٣}}$

$\cos\left(٢x + \dfrac{\pi}{٢}\right) = ٢\cos^{٢}\left(x + \dfrac{\pi}{٤}\right) - ١$

$\dfrac{٢}{٣} - \dfrac{٣}{٣} = -\dfrac{١}{٣}$

$-\sin ٢x = -\dfrac{١}{٣}$

$\sin ٢x = \dfrac{١}{٣}$

سوال ۹

$$(1+\cos 2x)(1+\cos 4x)(1+\cos 8x)=\frac{1}{8}$$

$$8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{8}$$

$\times\sin$

$$\sin\alpha\cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}$$

$$\frac{1}{2}\sin 2x\cos 2x \qquad \frac{1}{4}\sin 4x\cos 4x$$

$$\frac{1}{8}\sin 8x=\pm\frac{1}{8}$$

$$\sin\alpha(\sin(-8\alpha))$$

$$\sin 8\alpha=\sin\alpha$$

$$8\alpha=2k\pi+\alpha$$

$$8\alpha=2k\pi+\pi-\alpha \qquad \alpha=2k\pi-8\alpha$$

$$2k\frac{\pi+\pi}{9} \qquad \alpha=2k\pi+\pi+8\alpha$$

$$-7\alpha=2k\pi+\pi$$

$$\alpha=\frac{2k\pi}{9} \qquad \alpha=2k\frac{\pi+\pi}{-7}$$

max

$$\frac{2\pi}{9} \quad \frac{4\pi}{7} \quad \frac{7\pi}{9} \quad \frac{6\pi}{7}$$