سراسری

①

$$8\cos x = \frac{1}{\cos^2 x} \Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$$

$$\hookrightarrow \cos x \neq 0 \Rightarrow x \neq k\pi + \frac{\pi}{2}$$

۲ جواب

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②

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$$

$$(\cos x + 1)(8\cos^2 x - 13\cos x + 7)$$

(ریشه ندارد $\Delta < 0$)

$$\begin{array}{l|l} 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 & \cos x + 1 \\ \hline -8\cos^3 x - 8\cos^2 x & 8\cos^2 x - 13\cos x + 7 \\ -13\cos^2 x - 6\cos x + 7 & \\ +13\cos^2 x + 13\cos x & \\ 7\cos x + 7 & \end{array}$$

$$\cos x = -1$$

$$x = 2k\pi + \pi$$

در بازه گفته شده ۲ جواب دارد

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③

$$2\sin x(1 - 4\sin^2 x) = 2\sin x - 8\sin^3 x$$

$$4\sin^3 x - 3\sin x + 1 = 0$$

$$\begin{array}{l|l} 4\sin^3 x - 3\sin x + 1 & \sin x + 1 \\ \hline -4\sin^3 x - 4\sin^2 x & 4\sin^2 x - 4\sin x + 1 \\ -4\sin^2 x - 3\sin x + 1 & \\ +4\sin^2 x + 4\sin x & \\ \sin x + 1 & \end{array}$$

$$\Rightarrow (\sin x + 1)(2\sin x - 1)^2 = 0$$

$\sin x = -1 \Rightarrow x = 2k\pi - \frac{\pi}{2} \xrightarrow{\text{در بازه گفته شده}} x = \frac{3\pi}{2}$

$\sin x = \frac{1}{2} \longrightarrow x = \frac{\pi}{6}$

$\longrightarrow x = \frac{5\pi}{6}$

$$\frac{3\pi}{2} + \frac{\pi}{6} + \frac{5\pi}{6} = \frac{5\pi}{2}$$

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④

[illegible] $\Rightarrow \sin\left(\frac{\pi}{2} + 2x\right) = \dfrac{\cos\left(\frac{\pi}{4} + 4x\right)}{\sin 4x}$

$\sin\left(\frac{\pi}{2} + 2x\right) \neq 0$

$\Rightarrow \cos 2x \neq 0$

$\Rightarrow 2x \neq k\pi + \frac{\pi}{2} \Rightarrow x \neq \frac{k\pi}{2} + \frac{\pi}{4}$ ①

$\sin 4x \neq 0$

$4x \neq k\pi$

$\Rightarrow x \neq \frac{k\pi}{4}$ ②

$$\Rightarrow \frac{\pi}{2} + 2x = 2k\pi + 4x \Rightarrow 2x = \frac{\pi}{2} - 2k\pi \Rightarrow x = \frac{\pi}{4} - k\pi \quad ③$$

$$\frac{\pi}{2} + 2x = 2k\pi + \pi - 4x \Rightarrow 6x = 2k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{12} \quad ④$$

$$\tan\frac{2\pi}{3} = -\sqrt{3}$$

④، ③، ②، ①، بازه گفته شده در مسئله $\Rightarrow x = \frac{\pi}{12}$ یا $\frac{5\pi}{12} \Rightarrow d = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

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$$\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{3} \Rightarrow \cos\alpha - \sin\alpha = \frac{\sqrt{2}}{3}\times\frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$$

$$\frac{\sqrt{6}}{3}m - 2\sqrt{6}\sin^2\alpha = \sqrt{6} \Rightarrow 2\sqrt{6}\sin^2\alpha = -\frac{2\sqrt{6}}{3}m$$

$$\Rightarrow \sin^2\alpha = -\frac{2}{6}m \Rightarrow \frac{1}{3} = -\frac{2}{6}m \Rightarrow \boxed{m = -\frac{3}{2}}$$

$\sin^2\alpha = 2\sin\alpha\cos\alpha$

$$(\cos\alpha - \sin\alpha)^2 = 1 - \underbrace{2\sin\alpha\cos\alpha}_{\sin^2\alpha} = \frac{2}{3} \Rightarrow \sin^2\alpha = \frac{1}{3}$$

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$$= \left(\frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha\right)\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right) = 1$$

$$\frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha = 1 \Rightarrow \sin\left(\alpha + \frac{\pi}{6}\right) = 1 \Rightarrow \alpha + \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \Rightarrow \alpha = 2k\pi + \frac{\pi}{3}$$ ← جواب

$$\frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha = -1 \Rightarrow \sin\left(\alpha + \frac{\pi}{6}\right) = -1 \Rightarrow \alpha + \frac{\pi}{6} = 2k\pi - \frac{\pi}{2}$$

$$\Rightarrow \alpha = 2k\pi - \frac{2\pi}{3}$$ ← جواب

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$$\frac{1}{2}\sin\alpha + \frac{\sqrt{3}}{2}\cos\alpha = \frac{\sqrt{2}}{2} \Rightarrow \sin\left(\alpha + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)$$

$$\alpha + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \Rightarrow \alpha = 2k\pi - \frac{\pi}{12} \quad (-\frac{\pi}{12},\ \frac{23\pi}{12})$$

$$\alpha + \frac{\pi}{3} = 2k\pi + \pi - \frac{\pi}{4} \Rightarrow \alpha = 2k\pi - \frac{7\pi}{12} \quad (\frac{17\pi}{12},\ -\frac{7\pi}{12})$$

$$-\frac{\pi}{12} + \frac{23\pi}{12} - \frac{7\pi}{12} + \frac{17\pi}{12} = \frac{33\pi}{12} = \boxed{\frac{11\pi}{4}}$$

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$$-2\sin\alpha\cdot\cos\alpha = 1 \Rightarrow \sin 2\alpha = -\frac{1}{2}$$

$$2\alpha = 2k\pi - \frac{\pi}{6} \Rightarrow k\pi - \frac{\pi}{12} \quad (\frac{11\pi}{12},\ \frac{23\pi}{12})$$

$$2\alpha = 2k\pi + \pi - \frac{\pi}{6} \Rightarrow \alpha = k\pi + \frac{5\pi}{12} \quad (\frac{5\pi}{12},\ \frac{17\pi}{12})$$

$$\frac{11\pi}{12} + \frac{23\pi}{12} + \frac{5\pi}{12} + \frac{17\pi}{12} = \frac{56\pi}{12} = \boxed{\frac{14\pi}{3}}$$

$$\cos^2\alpha \times \cos^2 2\alpha \times \cos^2 4\alpha = \frac{1}{8}$$ (9)

$\max x = \pi$

$$|\cos\alpha \cos 2\alpha \cos 4\alpha| = \frac{1}{8} \xrightarrow[|\sin\alpha|]{\text{ضرب}} \frac{1}{8}|\sin 8\alpha| = \frac{1}{8}|\sin\alpha|$$

$\sin 8\alpha = \sin\alpha \Rightarrow 8\alpha = 2k\pi + \alpha \Rightarrow \alpha = \frac{2}{7}k\pi$ — $\frac{1}{7}$, $\frac{2}{7}$, $\frac{4}{7}$ — $8\alpha = 2k\pi + \pi - \alpha \Rightarrow \alpha = \frac{2}{9}k\pi + \frac{\pi}{9}$ — $\frac{\pi}{9}$, $\frac{3\pi}{9}$, $\frac{5\pi}{9}$, $\frac{7\pi}{9}$

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$\sin 8\alpha = \sin(-\alpha) \Rightarrow 8\alpha = 2k\pi - \alpha \Rightarrow \alpha = \frac{2}{9}k\pi - \frac{2\pi}{9}$ — $\frac{2\pi}{9}$, $\frac{4\pi}{9}$, $\frac{6\pi}{9}$ — $8\alpha = 2k\pi + \pi + \alpha \Rightarrow \alpha = \frac{2}{7}k\pi + \frac{\pi}{7}$ — $\frac{\pi}{7}$, $\frac{3\pi}{7}$, $\frac{5\pi}{7}$, π

(10)

$$\frac{\sin^2 x}{\cos^2 x} \times \frac{\sin x}{\cos x} = 1 \Rightarrow \sin^2 x \sin x - \cos^2 x \cos x = 0$$

$$\Rightarrow -\cos 3x = 0 \Rightarrow 3x = k\pi + \frac{\pi}{2}$$

$$x = \frac{k\pi}{3} + \frac{\pi}{6}$$

$$\frac{9\pi}{6} + \frac{12\pi}{6} = \frac{11\pi}{6}$$