۱۷/۵

سارنا ستاری

① $8\cos x = \dfrac{1}{\cos^2 x} \Rightarrow \cos^3 x = \dfrac{1}{8} \Rightarrow \cos x = \dfrac{1}{2} \Rightarrow x = 2k\pi \pm \dfrac{\pi}{3}$

$\hookrightarrow \cos x \neq 0 \Rightarrow x \neq k\pi + \dfrac{\pi}{2}$

۲ جواب

۲

---

② $\cos 3x = 4\cos^3 x - 3\cos x$

$\sin^2 x = 1 - \cos^2 x$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$(\cos x + 1)(8\cos^2 x - 13\cos x + 7)$ (ریشه ندارد $\Delta < 0$)

$$\begin{array}{l|l} 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 & \cos x + 1 \\ \hline -8\cos^3 x - 8\cos^2 x & 8\cos^2 x - 13\cos x + 7 \\ \hline -13\cos^2 x - 6\cos x + 7 & \\ +13\cos^2 x + 13\cos x & \\ \hline 7\cos x + 7 & \end{array}$$

$\cos x = -1$

$x = 2k\pi + \pi$

در بازه گفته شده ۲ جواب دارد

۲

---

③ $2\sin x\,(1 - 4\sin^2 x) = 3\sin x - 4\sin^3 x$

$4\sin^3 x - 3\sin x + 1 = 0$

$\Rightarrow (\sin x + 1)(2\sin x - 1)^2 = 0$

$$\begin{array}{l|l} 4\sin^3 x - 3\sin x + 1 & \sin x + 1 \\ \hline -4\sin^3 x - 4\sin^2 x & 4\sin^2 x - 4\sin x + 1 \\ \hline -4\sin^2 x - 3\sin x + 1 & \\ +4\sin^2 x + 4\sin x & \\ \hline \sin x + 1 & \end{array}$$

$\sin x = -1 \Rightarrow x = 2k\pi - \dfrac{\pi}{2} \xrightarrow[\text{گفته شده}]{\text{در بازه}} x = \dfrac{3\pi}{2}$

$\sin x = \dfrac{1}{2} \longrightarrow x = \dfrac{\pi}{6}$

$\longrightarrow x = \dfrac{5\pi}{6}$

$\dfrac{3\pi}{2} + \dfrac{\pi}{6} + \dfrac{5\pi}{6} = \dfrac{5\pi}{2}$

۲

---

④ بعد از طرفین وسطین $\Rightarrow \sin\left(\dfrac{\pi}{2} + 2x\right) = \underbrace{\cos\left(\dfrac{\pi}{2} + 4x\right)}_{-\sin 4x}$

$\sin\left(\dfrac{\pi}{2} + 2x\right) \neq 0 \Rightarrow \cos 2x \neq 0 \Rightarrow 2x \neq k\pi + \dfrac{\pi}{2} \Rightarrow x \neq \dfrac{k\pi}{2} + \dfrac{\pi}{4}$ ①

$\sin 4x \neq 0$

$4x \neq k\pi \Rightarrow x \neq \dfrac{k\pi}{4}$ ②

$\Rightarrow \dfrac{\pi}{2} + 2x = 2k\pi + 4x \Rightarrow 2x = \dfrac{\pi}{2} - 2k\pi \Rightarrow x = \dfrac{\pi}{4} - k\pi$ ③

$\dfrac{\pi}{2} + 2x = 2k\pi + \pi - 4x \Rightarrow 6x = 2k\pi + \dfrac{\pi}{2} \Rightarrow x = \dfrac{k\pi}{3} + \dfrac{\pi}{12}$ ④

۲

$\tan\dfrac{2\pi}{3} = -\sqrt{3}$

④,③,②,① , بازه گفته شده در مسئله $\Rightarrow x = \dfrac{\pi}{12}$ یا $\dfrac{5\pi}{12} \Rightarrow d = \dfrac{5\pi}{12} - \dfrac{\pi}{12} = \dfrac{\pi}{3}$

۵

$$\frac{\sqrt2}{2}\cos x-\frac{\sqrt2}{2}\sin x=\frac{\sqrt3}{3}\Rightarrow \cos x-\sin x=\frac{\sqrt3}{3}\times\frac{2}{\sqrt2}=\frac{2\sqrt6}{6}=\frac{\sqrt6}{3}$$

۱,۵

$$\frac{\sqrt6}{3}m-3\sqrt6\sin 2x=\sqrt6\Rightarrow 3\sqrt6\sin 2x=-\frac{2\sqrt6}{3}m$$

$$\Rightarrow \sin 2x=-\frac{2}{9}m\Rightarrow \frac{1}{3}=-\frac{2}{9}m\Rightarrow m=-\frac{3}{2}$$

×

$$\sin 2x=2\sin x\cos x$$

$$(\cos x-\sin x)^2=1-\underbrace{2\sin x\cos x}_{\sin 2x}=\frac{2}{3}\Rightarrow \sin 2x=\frac{1}{3}$$

۶

$$=\left(\frac{\sqrt3}{2}\sin x+\frac{1}{2}\cos x\right)\left(\frac{1}{2}\cos x+\frac{\sqrt3}{2}\sin x\right)=1$$

۲

۲ جواب

$$\frac{\sqrt3}{2}\sin x+\frac{1}{2}\cos x=1\Rightarrow \sin\left(x+\frac{\pi}{6}\right)=1 \quad x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2}$$

$$\Rightarrow x=2k\pi-\frac{\pi}{3}\rightarrow$$ ۱ جواب

$$\frac{\sqrt3}{2}\sin x+\frac{1}{2}\cos x=-1\Rightarrow \sin\left(x+\frac{\pi}{6}\right)=-1\rightarrow x+\frac{\pi}{6}=2k\pi-\frac{\pi}{2}$$

$$\Rightarrow x=2k\pi-\frac{2\pi}{3}\rightarrow$$ ۱ جواب

۱,۵ ۷

$$\frac{1}{2}\sin x+\frac{\sqrt3}{2}\cos x=\frac{\sqrt2}{2}\Rightarrow \sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}\right)$$

$$x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\Rightarrow x=2k\pi-\frac{\pi}{12} \quad \left(-\frac{\pi}{12},\ \frac{23\pi}{12}\right)$$

$$x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4}\Rightarrow x=2k\pi-\frac{7\pi}{12} \quad \left(-\frac{7\pi}{12},\ \frac{17\pi}{12}\right)$$

$$-\frac{\pi}{12}+\frac{23\pi}{12}-\frac{7\pi}{12}+\frac{17\pi}{12}=\frac{32\pi}{12}=\frac{8\pi}{3}$$

1,8 ۸

$$-2\sin x\cos x=1\Rightarrow \sin 2x=-\frac{1}{2}$$

$$2x=2k\pi-\frac{\pi}{6}\Rightarrow k\pi-\frac{\pi}{12} \quad \left(\frac{11\pi}{12},\ \frac{23\pi}{12}\right)$$

$$2x=2k\pi+\underbrace{\pi-\frac{\pi}{6}}_{\frac{5\pi}{6}}\Rightarrow x=k\pi+\frac{5\pi}{12} \quad \left(\frac{5\pi}{12},\ \frac{17\pi}{12}\right)$$

$$\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{5\pi}{12}+\frac{17\pi}{12}=\frac{56\pi}{12}=\frac{14\pi}{3}$$

$$2\cos^2\alpha \times 2\cos^2 2\alpha \times 2\cos^2 4\alpha = \frac{1}{8}$$ $max\,x = \pi$ 9

$$|\cos\alpha \cos2\alpha \cos4\alpha| = \frac{1}{8} \xrightarrow[|\sin\alpha|]{\text{ضرب در}} \frac{1}{8}|\sin 8\alpha| = \frac{1}{8}|\sin\alpha|$$ 1,5

$$\sin 8\alpha = \sin\alpha \Rightarrow 8\alpha = 2k\pi + \alpha \Rightarrow \alpha = \frac{2}{7}k\pi \quad (0,\ \tfrac{2}{7},\ \tfrac{4}{7},\ \tfrac{6}{7})$$
$$8\alpha = 2k\pi + \pi - \alpha \Rightarrow \alpha = \frac{2}{9}k\pi + \frac{\pi}{9} \quad (\pi,\ \tfrac{\pi}{9},\ \tfrac{\pi}{3},\ \tfrac{5\pi}{9},\ \tfrac{7\pi}{9})$$

$$\sin 8\alpha = \sin(-\alpha) \Rightarrow 8\alpha = 2k\pi - \alpha \Rightarrow \alpha = \frac{2}{9}k\pi - \frac{8\pi}{9} \quad (0,\ \tfrac{2\pi}{9},\ \tfrac{4\pi}{9},\ \tfrac{6\pi}{9})$$
$$8\alpha = 2k\pi + \pi + \alpha \Rightarrow \alpha = \frac{2}{7}k\pi + \frac{\pi}{7} \quad (\tfrac{\pi}{7},\ \tfrac{3\pi}{7},\ \tfrac{5\pi}{7},\ \pi)$$

10

$$\frac{\sin 3x}{\cos 3x} \times \frac{\sin x}{\cos x} = 1 \Rightarrow \sin3x\sin x - \cos3x\cos x = 0$$ 1,5

$$\Rightarrow -\cos4x = 0 \Rightarrow 4x = k\pi + \frac{\pi}{2}$$
$$x = \frac{k\pi}{4} + \frac{\pi}{8}$$
$$\frac{9\pi}{8} + \frac{13\pi}{8} = \frac{11\pi}{4}$$

$$\tan 3x = \frac{1}{\tan x} = \cot x \to \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$ 10

$$3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$

$$\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt2}{2}(\cos x - \sin x) = \frac{\sqrt3}{3} \to \cos x - \sin x = \frac{\frac{\sqrt3}{3}}{\frac{\sqrt2}{2}}$$ 5

$$\cos x - \sin x = \frac{\sqrt2}{\sqrt3} \xrightarrow{\text{گویا}} \frac{\sqrt6}{3} \to \boxed{\cos x - \sin x = \frac{\sqrt6}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \to \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt6(\sin 2x) = \sqrt6 \to \frac{\sqrt6}{3}m - 3\sqrt6\left(\frac{1}{3}\right) = \sqrt6$$

$$\frac{\sqrt6}{3}m - \sqrt6 = \sqrt6 \to \frac{m\sqrt6}{3} = 2\sqrt6 \to \boxed{m = 6}$$

(۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

---

(۸)

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} {}^{1}\; k=0,1 \longrightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ {}_{2}\; k=1,2 \longrightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

$$1+\cos 2x = 2\cos^2 x$$

$$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \rightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha\neq 0]{\times\sin\alpha}$$ ★

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\text{MAX } A} k=4 \rightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\text{MAX } A} k=2 \rightarrow x = \frac{5\pi}{7} \qquad \rightarrow \boxed{\text{MAX } A = \frac{8\pi}{9}}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\text{MAX } A} k=3 \rightarrow x = \frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!