دوازدهم ریاضی مهرنیا ریاضی — تکلیف شماره ۱۷ — ۱۴,۵

۱ (۱,۵)

$8\cos x - \tan^2 x = 1$

$8\cos x = \tan^2 x + 1 \qquad 8\cos x = \frac{1}{\cos^2 x} \qquad 8\cos^3 x = 1 \qquad \cos^3 x = \frac{1}{8} \qquad \cos x = \frac{1}{2}$

جواب ②

۲ (۲)

$5\sin^2(x) + 2\cos(3x) = -2$

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) + 2 = 0 \qquad 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$a + c = b + d$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = (\cos x + 1)(f)$ $\qquad f = 8\cos^2 x - 13\cos x + 7$

$(\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0 \longrightarrow \cos x = -1$ $\quad \pi,\ -\pi$ جواب ①

۳ (۲)

$2\sin x \cos 2x + \sin x = 1$

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \qquad 3\sin x - 4\sin^3 x = 1 \qquad \sin 3x = 1$

$3x = 2k\pi + \frac{\pi}{2} \qquad x = \frac{2k\pi}{3} + \frac{\pi}{6} \qquad \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{5\pi}{2}$

$3x = 2k\pi + \pi - \frac{\pi}{2}$ ✗

۴ (۱,۵)

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \Longrightarrow \cos 2x = \sin 4x$

$\sin 4x = \sin(\frac{\pi}{2} - 2x)$ $\quad 4x = 2k\pi + \frac{\pi}{2} - 2x$ ، $\quad 4x = 2k\pi + \frac{\pi}{2}$

ادامه؟

۵ (۲)

$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \qquad \cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

$\cos(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \qquad \cos x - \sin x = \frac{\sqrt{6}}{3}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = (\frac{\sqrt{6}}{3})^2 \qquad \sin 2x = \frac{1}{3}$ ✓

$\frac{\sqrt{6}}{3}m - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \qquad \frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \qquad \frac{\sqrt{6}}{3}m = 2\sqrt{6} \qquad (m = 6)$

۶ (۰,۷۵)

$\sin(x + \frac{\pi}{6})\cos(\frac{\pi}{3} - x) = 1$

$x + \frac{\pi}{6} + \frac{\pi}{3} - x = \frac{\pi}{2}$

Scanned with CamScanner

۷

$\sin x + \sqrt{3}\cos x = \sqrt{2}\quad [-\pi, \pi]$

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}\qquad \cos\frac{\pi}{3}\sin x + \sin\frac{\pi}{3}\cos x = \frac{\sqrt{2}}{2}\qquad \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$

$x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4}\qquad x = 2k\pi - \frac{\pi}{12}\qquad \frac{23\pi}{12} - \frac{\pi}{12} + \frac{2\pi}{12} = \frac{9\pi}{6}$ ✓ ۲

$x + \frac{\pi}{3} = 2k\pi + \pi - \frac{\pi}{4}\qquad x = 2k\pi + \frac{2\pi}{12}$

۸

$2\sin x \sin\left(\frac{\pi}{2} - x\right) = 1\qquad -2\sin x\cos x = 1\qquad -\sin 2x = 1$

$\sin 2x = -\frac{1}{2}$ → $2x = 2k\pi - \frac{\pi}{6}\qquad x = k\pi - \frac{\pi}{12}$

→ $2x = 2k\pi - \pi + \frac{\pi}{6}\qquad x = k\pi - \frac{2\pi}{12}$

۱/۵

$S = \{\}$

ادامه؟

۹

$(1+\cos(2x))(1+\cos(4x))(1+\cos(8x)) = \frac{1}{8}\qquad [0, \pi]$

$(2\cos^2 x)(2\cos^2 2x)(2\cos^2 4x) = \frac{1}{8}\qquad (\cos^2 x)(\cos^2 2x)(\cos^2 4x) = \frac{1}{64}$

۱/۲۵

$\cos x \cdot \cos 2x \cdot \cos 4x = \frac{1}{8}\qquad \times\frac{\sin x}{\sin x}\qquad \frac{\sin 8x}{8\sin x} = \frac{1}{8}$ ✓

$\sin 8x = \sin x$

max?

۱۰

$\tan 2x \tan x = 1\qquad \tan 2x = \cot x\qquad \tan 2x = \tan\left(\frac{\pi}{2} - x\right)$

$2x = k\pi + \frac{\pi}{2} - x\qquad x = \frac{k\pi}{3} + \frac{\pi}{6}$

$\frac{9\pi}{6} + \frac{11\pi}{6} + \frac{13\pi}{6} + \frac{15\pi}{6} = \frac{48\pi}{6} = 8\pi$ ✓ ۲

① $8\cos x = 1+\tan^2 x \rightarrow 8\cos x = \frac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1$

$\cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi}$ $\boxed{x = \frac{\pi}{3} \text{ یا } \frac{5\pi}{3}}$

---

④ $\begin{cases} \sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x \end{cases} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \rightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$

$\cos 2x - 2\sin 2x \cos 2x = 0 \rightarrow \cos 2x(1-2\sin 2x) = 0$

$\cos 2x = 0$ ✗

$\sin 2x = \frac{1}{2}$ ✓

* اگر $\cos 2x = 0$ باشد مضرج عبارت اولیه صفر می‌شود که جزء قابل قبول است!

$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12} \text{ یا } k\pi + \frac{5\pi}{12}$

$\xrightarrow{0 \le x \le \pi} k = 0 \rightarrow x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12} \rightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$

$\alpha = \frac{\pi}{3} \rightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$

---

⑤ $\cos(x-\frac{\pi}{2}) = \cos(\frac{\pi}{2}-x)$

$\frac{\pi}{2} - x + x + \frac{\pi}{4} = \frac{3\pi}{4}$ $\rightarrow \cos(\frac{\pi}{2}-x) = \sin(x+\frac{\pi}{4})$

$\sin^2(x+\frac{\pi}{4}) = 1 \rightarrow \sin(x+\frac{\pi}{4}) = \pm 1 \rightarrow x+\frac{\pi}{4} = k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{4} \xrightarrow{0 \le x \le 2\pi} k=0 \rightarrow x = \frac{\pi}{4}$ ، $k=1, x = \frac{5\pi}{4}$

معادله ۲ جواب دربازه دارد

(۹)

$1+\cos 2x = 2\cos^2 x$

$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$

$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{}} \cos\alpha\cos 2\alpha\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha\neq 0]{\times \sin\alpha}$

$(\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha})\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}(\underbrace{\sin 2\alpha\cos 2\alpha}_{\frac{1}{2}\sin 4\alpha})\cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2}\times\frac{1}{2}(\underbrace{\sin 4\alpha\cos 4\alpha}_{\frac{1}{2}\sin 8\alpha}) = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{8}\sin 8\alpha = \pm\frac{\sin\alpha}{8}$

$\sin 8\alpha = \pm\sin\alpha \rightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$

$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX\,A} k=3 \rightarrow x=\frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX\,A} k=4 \rightarrow x=\frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX\,A} k=2 \rightarrow x=\frac{5\pi}{7}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX\,A} k=3 \rightarrow x=\frac{7\pi}{9}$

$\rightarrow \boxed{MAX\,A = \frac{8\pi}{9}}$

* A نمی‌تواند برابر خود π باشد چون طبق * sinα را مخالف صفر در نظر گرفته‌ایم!

---

(۸)

$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$

$4\sin x(-\cos x)=1 \rightarrow -2(2\sin x\cos x)=1 \rightarrow \sin 2x = -\frac{1}{2}$

$2x = 2k\pi-\frac{\pi}{6} \ \text{یا}\ 2k\pi+\frac{7\pi}{6} \rightarrow x = k\pi-\frac{\pi}{12}\ (k=2) \ \text{یا}\ k\pi+\frac{7\pi}{12}\ (k=1)$

$0\le x\le 2\pi \xrightarrow{} \begin{cases} k=0,1 \rightarrow x=\frac{7\pi}{12},\frac{19\pi}{12} \\ k=1,2 \rightarrow x=\frac{11\pi}{12},\frac{23\pi}{12} \end{cases}$

$S = \frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$