۱- مینیاوونی

$\wedge\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \rightarrow \wedge\cos^3 x = 1 \rightarrow \cos^3 x = \frac{1}{\wedge}$

$\cos x = \frac{1}{2}$

$\cos x = \cos\frac{\pi}{3} \rightarrow$

$x = 2k\pi \pm \frac{\pi}{3}$

$0 \le \sin^2(x) \le 1 \Rightarrow \quad 0 \le \cos^2 x \le 1 \Rightarrow$

$-1 \le \cos(2x) \le 1 \Rightarrow \quad -2 \le 2\cos(2x) \le 2$

$-2 \le \cos^2 x + 2\cos(2x) \le 3$

$\Rightarrow \sin^2(x) = 0 \Rightarrow x = k\pi$

$\cos(2x) = -1 \Rightarrow 2x = 2k\pi + \pi$

$x = \frac{2k\pi}{2} + \frac{\pi}{2}$

↳ min

$\Rightarrow x = -\pi \text{ و } \pi$

$\ldots \sin x - 3\cos x - 4\cos^3 x + \ldots$

-3

$2\sin x(1-2\sin^2 x)+\sin x=1 \Rightarrow 2\sin x-4\sin^3 x+\sin x=1$

$4\sin^3 x - 3\sin x+1=0 \to \sin x=t \quad 4t^3-3t+1=0 \Rightarrow t=-1$

$$\begin{array}{l|l} 4t^3-3t+1 & t+1 \\ 4t^3+4t^2 & 4t^2-4t+1 \\ \hline -4t^2-3t+1 & \\ -4t^2-4t & \\ \hline t+1 & \end{array}$$

$(t+1)(2t-1)^2$

$\sin x=-1 \to x=2k\pi-\frac{\pi}{2}$

$\sin x=\frac{1}{2} \to x=2k\pi+\frac{\pi}{6}$

$x=2k\pi+\frac{5\pi}{6}$

$\to \frac{3\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{5\pi}{2}$

-4

$\frac{1}{\underbrace{\sin(\frac{\pi}{2}+2x)}_{\cos 2x}}+\frac{1}{\underbrace{\cos(\frac{\pi}{2}+4x)}_{-\sin 4x}}=\frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0$

$\frac{\sin 4x-\cos 2x}{\cos 2x\sin 4x}=0$

$2\sin 2x\cos 2x$ ~~$-\cos 4x$~~

$\cos 2x(2\sin 2x-1)=0 \to \cos 2x=0 \to$ ✗

$\searrow \sin 2x=\frac{1}{2} \to 2x=2k\pi+\frac{\pi}{6}$

$2x=2k\pi+\frac{5\pi}{6}$

$\to k\pi+\frac{\pi}{12} \quad x=k\pi+\frac{5\pi}{12}$

$x=\frac{\pi}{12}, \frac{5\pi}{12} \to \frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \to \tan\frac{2\pi}{3}=-\sqrt{3}$

۵-

$\cos(x-\frac{\pi}{4}) = \cos(x+\frac{\pi}{4}-\frac{\pi}{2}) = \cos(\frac{\pi}{2}-(x+\frac{\pi}{4})) = \sin(x+\frac{\pi}{4})$ ۶-

$\rightarrow \sin^2(x+\frac{\pi}{4}) = 1 \rightarrow \sin(x+\frac{\pi}{4}) \geq 1$

$\sin(x+\frac{\pi}{4}) \leq -1$

$x+\frac{\pi}{4} = k\pi+\frac{\pi}{2}$

$x = \frac{\pi}{4}$ ، $\frac{5\pi}{4}$

$x = k\pi+\frac{\pi}{4} \rightarrow$

۷-

$2\sin(x+a) \rightarrow 2\sin(x+\frac{\pi}{4}) = \sqrt{2} \rightarrow \sin(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\tan\alpha = \sqrt{2}$

$x+\frac{\pi}{4} = 2k\pi+\frac{\pi}{3} \rightarrow x = 2k\pi+(-\frac{\pi}{12})$

$x+\frac{\pi}{4} = 2k\pi+\frac{2\pi}{3}$

$x = -\frac{\pi}{12}$ ، $\frac{5\pi}{12}$ ، $\frac{13\pi}{12}$

۸-

$2\sin x(-\cos x) = 1 \rightarrow -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6})$

$2x = 2k\pi-\frac{\pi}{6} \rightarrow x = k\pi-\frac{\pi}{12}$ $\quad 2x = 2k\pi+\frac{7\pi}{6} \rightarrow x = k\pi+\frac{7\pi}{12}$

$x = \frac{11\pi}{12}$ ، $\frac{17\pi}{12}$ ، $\frac{19\pi}{12}$ ، $\frac{23\pi}{12}$

۹-

$\cos x - \sin x = \frac{\sqrt{2}}{2}$

$\frac{\frac{\sqrt{2}}{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \sqrt{2} \rightarrow \sin 2x = -\frac{1}{2}$

$-\frac{1}{2} = \frac{1}{2} \rightarrow x = -\frac{\pi}{12}$

۱۰- $(\cos x-\sin x)^2 = 1-\sin 2x = \frac{2}{3} \rightarrow \sin 2x = \frac{1}{3}$

$\tan x = \cot(4x) \rightarrow \tan x = \tan\left(\frac{\pi}{2} - 4x\right)$

-10

$\rightarrow x = k\pi + \frac{\pi}{2} - 4x \Rightarrow x = \frac{k\pi}{5} + \frac{\pi}{10} \rightarrow x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}$