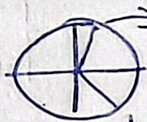
۱۵,۵

۱ـ مبینا فونی

$8\cos$

$8\cos x = 1+\tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 8\cos^3 x = 1 \Rightarrow \cos^3 x = \frac{1}{8}$

$\cos x = \frac{1}{2}$



$\frac{\pi}{3}$

$\frac{5\pi}{3}$

$\cos x = \cos\frac{\pi}{3} \Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

۲

-۲

$0 \leq \sin^۲(x) \leq ۱ \Rightarrow 0 \leq ۵\sin^۲ x \leq ۵ \Rightarrow$ ۲

$-۱ \leq \cos(۳x) \leq ۱ \Rightarrow -۲ \leq ۲\cos(۳x) \leq ۲$

$-۲ \leq ۵\sin^۲ x + ۲\cos(۳x) \leq ۷ \Rightarrow \sin^۲(x) = 0 \Rightarrow x = k\pi$

min

و

$\cos ۳x = -۱ \Rightarrow ۳x = ۲k\pi + \pi$

$x = \frac{۲k\pi}{۳} + \frac{\pi}{۳}$

$\Rightarrow x = -\pi$ و π

$$2\sin x(1-2\sin^2 x)+\sin x=1 \Rightarrow 2\sin x-4\sin^3 x+\sin x=1$$

$$4\sin^3 x-3\sin x+1=0 \to \sin x=t \quad 4t^3-3t+1=0 \Rightarrow t=-1$$

$$\begin{array}{r|l} 4t^3-3t+1 & t+1 \\ \hline 4t^3+4t^2 & 4t^2-4t+1 \\ -4t^2-3t+1 & \\ -4t^2-4t & \\ t+1 & \end{array}$$

$$(t+1)(2t-1)^2$$

$$\sin x=-1 \Rightarrow x=2k\pi-\frac{\pi}{2}$$

$$\sin x=\frac{1}{2} \Rightarrow x=2k\pi+\frac{\pi}{6}$$

$$x=2k\pi+\frac{5\pi}{6}$$

$$\Rightarrow \frac{3\pi}{2}+\frac{\pi}{6}+\frac{5\pi}{6}=\frac{5\pi}{2}$$

$$\frac{1}{\underbrace{\sin(\frac{\pi}{2}+2x)}_{\cos 2x}}+\frac{1}{\underbrace{\cos(\frac{\pi}{2}+4x)}_{-\sin 4x}}=\frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0$$

$$\frac{\sin 4x-\cos 2x}{\cos 2x\sin 4x}=0$$

$$\cos 2x(2\sin 2x-1)=0 \Rightarrow \cos 2x=0 \to \times$$

$$\sin 2x=\frac{1}{2} \Rightarrow 2x=2k\pi+\frac{\pi}{6}$$

$$2x=2k\pi+\frac{5\pi}{6}$$

$$\Rightarrow k\pi+\frac{\pi}{12} \qquad x=k\pi+\frac{5\pi}{12}$$

$$x=\frac{\pi}{12},\ \frac{5\pi}{12} \Rightarrow \frac{5\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3} \Rightarrow \tan\frac{2\pi}{3}=-\sqrt{3}$$

$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$ ۵-

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$

$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$

$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$

$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$

۶-

$\cos(x-\frac{\pi}{3}) = \cos(x+\frac{\pi}{6}-\frac{\pi}{2}) = \cos(\frac{\pi}{2}-(x+\frac{\pi}{6})) = \sin(x+\frac{\pi}{6})$

$\rightarrow \sin^2(x+\frac{\pi}{6}) = 1 \rightarrow \sin(x+\frac{\pi}{6}) = 1$

$\sin(x+\frac{\pi}{6}) = -1$

2

$x+\frac{\pi}{6} = k\pi+\frac{\pi}{2}$

$x = k\pi+\frac{\pi}{3} \rightarrow$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

۷-

$2\sin(x+\alpha) \rightarrow 2\sin(x+\frac{\pi}{3}) = \sqrt{2} \rightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$

$\tan\alpha = \sqrt{3}$

1,5

$x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi+(-\frac{\pi}{12})$

$x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4}$

S?

$x = -\frac{\pi}{12}, \frac{3\pi}{4}, \frac{23\pi}{12}$

۸-

$4\sin x(-\cos x) = 1 \rightarrow -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6})$ 1,5

$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$ $\quad 2x = 2k\pi+\frac{7\pi}{6} \rightarrow x = k\pi+\frac{7\pi}{12}$

$x = \frac{11\pi}{12}, \frac{23\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{12}$

S?

۹-

1

$\cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{6}}{3} \rightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\sin 2x = \sqrt{6} \rightarrow \sin 2x = -\frac{2}{9}m$

$-\frac{2}{9}m = \frac{1}{3} \rightarrow m = -\frac{3}{2}$

S: $(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{2}{3} \rightarrow \sin 2x = \frac{1}{3}$

$\tan x = \cot(3x) \rightarrow \tan x = \tan\left(\frac{\pi}{2} - 3x\right)$

1,5 -10

$\rightarrow x = k\pi + \frac{\pi}{2} - 3x \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8} \rightarrow x = \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

s ?

⑦ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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⑧

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$4\sin x(-\cos x) = 1 \longrightarrow -2(2\sin x\cos x) = 1 \longrightarrow \sin 2x = -\frac{1}{2}$$

$$2x = 2k\pi - \frac{\pi}{6} \text{ یا } 2k\pi + \frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi - \frac{\pi}{12}} \text{ یا } \overset{1}{k\pi + \frac{7\pi}{12}}$$

$$\xrightarrow{0 \le x \le 2\pi} \begin{cases} (1)\ k=0,1 \longrightarrow x = \frac{7\pi}{12}, \frac{19\pi}{12} \\ (2)\ k=1,2 \longrightarrow x = \frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S = \frac{7\pi}{12} + \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = \boxed{5\pi}$$

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⑩

$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9+11+13+15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$

$$1+\cos 2* = 2\cos^2 *$$

$$\left\{\begin{array}{l} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{array}\right\} ? \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\frac{1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \; \star}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \rightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \left\{\begin{array}{l} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{array}\right.$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow n = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow n = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow n = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow n = \frac{7\pi}{9}$$

$$\rightarrow \boxed{\max A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!