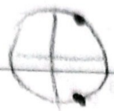
نازنین صفری — تکلیف ۳ — ۱۰ و ۹

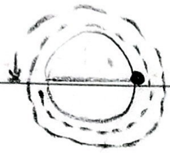
۱ـ $۸\cos x - \tan^{۲} x = ۱ \Rightarrow ۸\cos x = ۱ + \tan^{۲} x$

$\Rightarrow ۸\cos x = \frac{۱}{\cos^{۲} x} \xrightarrow{\cos x \neq ۰} ۸\cos^{۳} x = ۱ \Rightarrow \cos x = \frac{۱}{۲}$

$x = ۲k\pi \pm \frac{\pi}{۳}$  ۲ جواب

۲ـ $۵\sin^{۲} x + ۲\cos ۳x = -۲ \Rightarrow ۵(۱ - \cos^{۲} x) + ۲(۴\cos^{۳} x - ۳\cos x) = -۲$

$۸\cos^{۳} x - ۵\cos^{۲} x - ۶\cos x + ۷ = ۰$ جمع ضریب‌ها برابر صفر

$\cos x = +۱ \rightarrow$  ۲ جواب — و در صورت تقسیم، ریشه دوم $\Delta < ۰$ دارد.

۳ـ $۲\sin x \cos ۲x + \sin x = ۱ \rightarrow \sin x(۲\cos ۲x + ۱) = ۱$

$\sin x(۲(۲\cos^{۲} x - ۱) + ۱) = ۱ \rightarrow \sin x(۴\cos^{۲} x - ۱) = ۱$

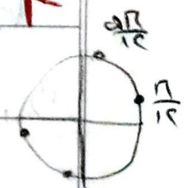
$\sin x(۳ - ۴\sin^{۲} x) = ۱ \rightarrow ۴\sin^{۳} x - ۳\sin x + ۱ = ۰$

$\rightarrow \begin{cases} \sin x = -۱ \rightarrow ۲k\pi - \frac{\pi}{۲} \\ \sin x = \frac{۱}{۲} \rightarrow x = ۲k\pi + \frac{\pi}{۶} ,\ ۲k\pi + \frac{۵\pi}{۶} \end{cases}$

$\Rightarrow \frac{\pi}{۶} + \frac{۵\pi}{۶} + \frac{۳\pi}{۲} = \frac{۵\pi}{۲}$

۴ـ $\frac{۱}{\cos ۲x} - \frac{۱}{\sin ۴x} = ۰ \rightarrow \cos ۲x = \overbrace{\sin ۴x}^{۲\cos ۲x \sin ۲x} \rightarrow ۱ = ۲\sin ۲x$

$\rightarrow \sin ۲x = \frac{۱}{۲} \rightarrow \begin{cases} x = k\pi + \frac{\pi}{۱۲} \\ x = k\pi + \frac{۵\pi}{۱۲} \end{cases}$



$\frac{۵\pi}{۱۲} - \frac{\pi}{۱۲} = \frac{\pi}{۳} \rightarrow \tan \frac{۲\pi}{۳} = -\sqrt{۳}$

$\frac{1}{\sqrt{2}}$

$\cos x - \sin x = \sqrt{2}\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{\sqrt{2}}$ ، $(\cos x - \sin x)^2 = 1 - \sin 2x$ ω

$\Rightarrow \sin 2x = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow m\left(\frac{\sqrt{2}}{\sqrt{2}}\right) - 2\sqrt{4} \times \frac{1}{3} = \sqrt{4}$

$\Rightarrow m \times \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{4} \rightarrow m = 4$

$\sin\left(x + \frac{\pi}{4}\right)\cos\left(\frac{\pi}{3} - x\right) = 1 \xrightarrow{\text{ضرب}} \sin^2\left(x + \frac{\pi}{4}\right) = 1$ ـ۶

$\sin\left(x + \frac{\pi}{4}\right) = 1 \rightarrow x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$

$\sin\left(x + \frac{\pi}{4}\right) = -1 \rightarrow x + \frac{\pi}{4} = 2k\pi - \frac{\pi}{2} \rightarrow x = 2k\pi - \frac{3\pi}{3}$

$-1, 0, 1$

$\sin x + \sqrt{3}\cos x = \sqrt{2} \rightarrow 2\sin\left(x + \frac{\pi}{3}\right) = \sqrt{2}$ ـ۷

$\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \rightarrow \begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \pi - \frac{\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12} \end{cases}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12} \xrightarrow{\text{جمع}} \frac{9\pi}{4}$

$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1 \rightarrow 4\sin x \times -\cos(-x) = 1$ ـ۸

$-2(2\sin x \cos x) = 1 \rightarrow \sin 2x = -\frac{1}{2} \rightarrow \begin{cases} 2x = 2k\pi + \frac{7\pi}{4} \\ 2x = 2k\pi + \pi - \frac{7\pi}{4} \end{cases}$

$x = k\pi + \frac{7\pi}{12} \xrightarrow{[0, 2\pi]} \frac{7\pi}{12}, \frac{19\pi}{12}$

$x = k\pi + \frac{11\pi}{12} \xrightarrow{[0, 2\pi]} \frac{11\pi}{12}, \frac{23\pi}{12}$ $\Big\}$ 5π

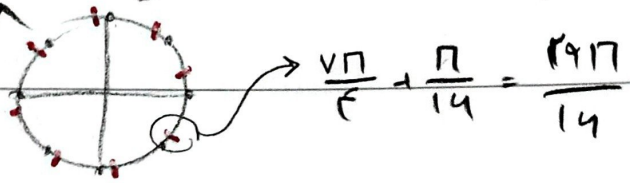
$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$ -9

$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=8(\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha)^2=\frac{1}{8}$

$\cos\alpha\cos 2\alpha\cos 4\alpha=\frac{1}{8}\times\frac{\sin 8\alpha}{\sin\alpha}\cos 4\alpha\cos 8\alpha=\frac{1}{16}\sin 8\alpha\cos 8\alpha$

$=\frac{1}{16}\sin 16\alpha=\frac{1}{16}\longrightarrow \sin 16\alpha=1\longrightarrow 16\alpha=2k\pi+\frac{\pi}{2}$

$\alpha=\frac{k\pi}{8}+\frac{\pi}{32}$

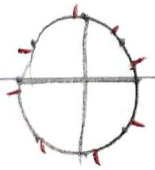


$\frac{7\pi}{8}+\frac{\pi}{32}=\frac{29\pi}{32}$

$\tan^2 3x=\frac{1}{\tan x}=\cot x=\tan\left(\frac{\pi}{2}-x\right)\longrightarrow 3x=k\pi+\frac{\pi}{2}-x$ -10

$\longrightarrow 4x=k\pi+\frac{\pi}{2}\longrightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$

k	1	2	3	4
x	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$	$\frac{9\pi}{8}$



$4\times 2=8\pi$