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تاج‌گردن

(۱)

$$8\cos\alpha = \frac{1+\tan^2\alpha}{\frac{1}{\cos^2\alpha}} \rightarrow 8\cos^3\alpha = 1 \rightarrow \cos\alpha = \frac{1}{2}$$ (۲)

۲ جواب

(۲)

$$5\sin^2\alpha + 2\sin^2\alpha + 2\cos^2\alpha + 2\cos 2\alpha = 5$$

$$5(1-\cos^2 x) + 2(2\cos^2 x - 2\cos x) + 2 = 0$$ (۲)

π ، $-\pi$ ← ۲ جواب

(۳)

$$2\sin\cos 2x + \sin x = 1 \rightarrow 2\sin x(1-2\sin^2 x) + \sin x = 1$$

$$3\sin x - 4\sin^3 x = 1 \rightarrow \sin 3x = 1 \rightarrow \frac{\pi}{6}, \cancel{\frac{5\pi}{6}}, \frac{9\pi}{6} \rightarrow \frac{5\pi}{3}$$ (۲)

(۴)

$$\frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \longrightarrow \cos 2x = \sin 4x$$ (۱)

$$\sin 4x = \sin\left(\frac{\pi}{2} - 2x\right) \rightarrow \frac{2k\pi + \frac{\pi}{2}}{6}, \quad \rightarrow \frac{2k\pi + \frac{\pi}{2}}{2}$$

ادامه؟

(۵)

$$\tan 2x \tan x = 1 \quad \tan 2x = \cot x \quad \tan 2x = \tan\left(\frac{\pi}{2} - x\right)$$

$$2x = k\pi + \frac{\pi}{2} - x \rightarrow x = \frac{k\pi}{3} + \frac{\pi}{6} \rightarrow \frac{9\pi}{8} + \frac{11\pi}{8} + \cancel{\frac{13\pi}{8}} + \frac{15\pi}{8} = \frac{48\pi}{8} = 6\pi$$ (۲)

(۶)

$$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1 \longrightarrow -4\sin x\cos x = 1 \rightarrow -2\sin 2x = 1$$

$$\sin 2x = -\frac{1}{2} \rightarrow k\pi - \frac{\pi}{12}, \quad k\pi - \frac{5\pi}{12}$$ (۱)

(۷)

$$\sin\left(x + \frac{\pi}{6}\right)\cos\left(\frac{\pi}{3} - x\right) = 1 \rightarrow x + \frac{\pi}{6} + \frac{\pi}{3} - x = \frac{\pi}{2} \rightarrow \sin\left(x + \frac{\pi}{6}\right) = \pm 1$$ (۲)

$\left(k\pi + \frac{\pi}{2} - \frac{\pi}{6}\right)$ ۲ جواب

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$$\begin{cases} \sin(2x+\frac{\pi}{2}) = \cos 2x \\ \cos(\frac{\pi}{2}+4x) = -\sin 4x \end{cases} \longrightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0 \longrightarrow \frac{\cos 2x - \sin 4x}{\cos 2x(-\sin 4x)} = 0$$

$$\cos 2x - 2\sin 2x\cos 2x = 0 \longrightarrow \cos 2x(1-2\sin 2x) = 0 \longrightarrow \begin{cases} \cos 2x = 0 \quad \times \\ \sin 2x = \frac{1}{2} \quad \checkmark \end{cases}$$

* اگر $\cos 2x = 0$ باشد مخرج عبارت اولیه صفر می‌شود که غیر قابل قبول است!

$$\sin 2x = \frac{1}{2} \longrightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \longrightarrow x = k\pi + \frac{\pi}{12} \text{ یا } k\pi + \frac{5\pi}{12}$$

$$\xrightarrow{0 \le x \le \pi} k=0 \longrightarrow x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12} \longrightarrow \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3} \longrightarrow \tan(2\alpha) = \tan(\frac{2\pi}{3}) = \boxed{-\sqrt{3}}$$

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$$\cos(x+\frac{\pi}{4}) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}(\frac{1}{3}) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m=6}$$

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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$$\sin\left(\frac{3\pi}{2}-x\right)=-\cos x$$

$$4\sin x(-\cos x)=1 \longrightarrow -2(2\sin x\cos x)=1 \longrightarrow \sin 2x=-\frac{1}{2}$$

$$2x=2k\pi-\frac{\pi}{6} \text{ یا } 2k\pi+\frac{7\pi}{6} \longrightarrow x=\overset{2}{k\pi-\frac{\pi}{12}} \text{ یا } \overset{1}{k\pi+\frac{7\pi}{12}}$$

$$\xrightarrow{0\leq x\leq 2\pi} \begin{cases} \overset{1}{k=0,1} \longrightarrow x=\frac{7\pi}{12},\ \frac{19\pi}{12} \\ \underset{2}{k=1,2} \longrightarrow x=\frac{11\pi}{12},\ \frac{23\pi}{12} \end{cases}$$

$$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

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$$1+\cos 2x=2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha=2\cos^2\alpha \\ 1+\cos 4\alpha=2\cos^2 2\alpha \\ 1+\cos 8\alpha=2\cos^2 4\alpha \end{cases} \longrightarrow 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha=\frac{\pm 1}{8} \xrightarrow[\sin\alpha\neq 0]{\times\sin\alpha\ *}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha=\pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha=\pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha}=\pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha=\pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha=\pm\sin\alpha \longrightarrow \begin{cases} 8\alpha=2k\pi+\alpha \text{ یا } 2k\pi+\pi-\alpha \\ 8\alpha=2k\pi-\alpha \text{ یا } 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha=\frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x=\frac{6\pi}{7}$$

$$\alpha=\frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x=\frac{8\pi}{9}$$

$$\alpha=\frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x=\frac{5\pi}{7}$$

$$\alpha=\frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x=\frac{7\pi}{9}$$

$$\longrightarrow \boxed{\max A=\frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!