B - حوالعصر حضر خورشید احمدی

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۱) $8\cos x - \tan^2 x = 1 \rightarrow 8\cos x = 1+\tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x}$

$\xrightarrow{\cos x \neq 0} \cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{x\in[0,2\pi]}$

$x : \left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\} \Rightarrow$ ۲ جواب دارد

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۲) $\underbrace{5\sin^2 x}_{\text{همیشه مثبت یا صفر}} + 2\cos 3x = -2 \Rightarrow 2\cos 3x \leq -2 \rightarrow \cos 3x \leq -1 \xrightarrow{-1\leq \cos x\leq 1}$ پس $\cos 3x = -1$

$\xrightarrow{\cos 3x = -1}$ و $\sin x = 0$

$\Rightarrow x = \{k\pi\} \cap \left\{\frac{2k\pi \pm \pi}{3}\right\} \overset{x\in[-\pi,\pi]}{\Rightarrow} x = \{-\pi, \pi\}$

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۳) $2\sin x \underbrace{\cos 2x}_{-2\sin^2 x+1} + \sin x = 1 \rightarrow \sin x(\underbrace{2 - \cancel{4\sin^2 x + 1}}_{(3-4\sin^2 x)}) = 1$

$\rightarrow 4\sin^3 x - 3\sin x - 1 = 0 \xrightarrow[ax^3+bx^2+cx+d]{a+b+c+d=0} \sin x = 1$ [illegible] حالت

$\Rightarrow (\sin x - 1)(4\sin^2 x + 4\sin x + 1) = 0 \rightarrow (\sin x - 1)(2\sin x + 1)^2 = 0$

$\Rightarrow \begin{cases} \sin x = -\frac{1}{2} \rightarrow 2k\pi + \frac{7\pi}{6}, \ 2k\pi - \frac{\pi}{6} \\ \sin x = 1 \rightarrow 2k\pi + \frac{\pi}{2} \end{cases} \xrightarrow{x\in[0,2\pi]} x = \left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$\boxed{S = \frac{7\pi}{2}}$

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۴) $\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = 0 \rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0$

$\rightarrow \frac{2\sin 2x - 1}{\sin 4x} = 0 \xrightarrow{\sin 4x \neq 0} \sin 2x = \frac{1}{2} \rightarrow 2x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \frac{5\pi}{6} \end{cases}$

$\Rightarrow x = \begin{cases} k\pi + \frac{\pi}{12} \\ k\pi + \frac{5\pi}{12} \end{cases} \xrightarrow{\sin 4x \neq 0} \begin{cases} x_1 = \frac{\pi}{12} \\ x_2 = \frac{5\pi}{12} \end{cases} \Rightarrow \overset{\alpha}{x_2 - x_1} = \frac{\pi}{3} \rightarrow \tan 2\alpha = \tan\frac{2\pi}{3} = \boxed{-\sqrt{3}}$

(۵

$$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \rightarrow -\sqrt{2}.m(\sin(x+\frac{\pi}{4})) - 3\sqrt{6}\sin 2x = \sqrt{6}$$

$$\rightarrow m(\sin(x+\frac{\pi}{4})) + 3\sqrt{3}\sin 2x = -\sqrt{3} \xrightarrow{\cos(x+\frac{\pi}{4}) = \frac{\sqrt{3}}{2}} \sin(\frac{\pi}{2} - x - \frac{\pi}{4}) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cancel{\pm} m\frac{\cancel{\sqrt{3}}}{2} + (-)3\cancel{\sqrt{3}}\sin 2x = \cancel{\pm}\cancel{\sqrt{3}} \rightarrow \frac{1}{2}m - 3\sin 2x = 1 \Rightarrow$$

$$\frac{m-3}{9} = \sin 2x \xrightarrow[\Rightarrow x = 2k\pi - \frac{\pi}{12},\ \frac{24k\pi - 5\pi}{12}]{\cos(x+\frac{\pi}{4}) = \frac{\sqrt{3}}{2}} \frac{m-3}{9} = -\frac{1}{2} \Rightarrow \boxed{m = -\frac{3}{2}}$$

(۶

$$\sin(x+\frac{\pi}{4}).\cos(x-\frac{\pi}{3}) = \sin(\pi-(x+\frac{\pi}{6})).\sin(\frac{\pi}{2}-(x-\frac{\pi}{3}))$$

$$\Rightarrow \sin(\frac{5\pi}{6}-x).\sin(\frac{5\pi}{6}-x) = 1 \Rightarrow \sin(\frac{5\pi}{6}-x) = \pm 1 \rightarrow$$

$$\sin(\frac{5\pi}{6}-x) = \sin(\frac{\pi}{2}) \rightarrow x = \frac{\pi - 6k\pi}{3}$$

$$\sin(\frac{5\pi}{6}-x) = \sin(\frac{3\pi}{2}) \rightarrow x = \frac{-2\pi - 6k\pi}{3}$$

$$\xrightarrow{x\in[0,2\pi]} \boxed{x = \{\frac{\pi}{3}, \frac{4\pi}{3}\}}$$

(۷

$$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div\frac{1}{2}} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \rightarrow$$

$$\cos(x-\frac{\pi}{6}) = \cos(\frac{\pi}{4}) \Rightarrow x - \frac{\pi}{6} = 2k\pi \pm \frac{\pi}{4} \Rightarrow x = \frac{24k\pi+5\pi}{12},\ \frac{24k\pi-\pi}{12}$$

$$\xrightarrow{x\in[-\pi,2\pi]} x = \{-\frac{\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12}\} \rightarrow \boxed{S = \frac{9\pi}{4}}$$

(۸

$$4\sin x \sin(\frac{3\pi}{2}-x) = -4\sin x\cos x = -2\sin 2x = 1$$

$$\rightarrow \sin 2x = -\frac{1}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = \frac{12k\pi - \pi}{12}$$

$$2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = \frac{12k\pi + 7\pi}{12}$$

$$\xrightarrow{x\in[0,2\pi]} x = \{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\}$$

$$\boxed{S = 5\pi}$$

(۹

$$(1+\underbrace{\cos 2a}_{2\cos^2 a})(1+\underbrace{\cos 4a}_{2\cos^2 2a})(1+\underbrace{\cos 8a}_{2\cos^2 4a}) = \frac{1}{8} \xrightarrow{\times \sin^2 a} \frac{1}{8}\sin^2 8a = \frac{1}{8}$$

$$\rightarrow \sin^2 8a = 1 \rightarrow \sin 8a = \pm 1 \rightarrow \begin{cases} 8a = 2k\pi + \frac{\pi}{2} \rightarrow a = \frac{k\pi}{4} + \frac{\pi}{16} \\ 8a = 2k\pi + \frac{3\pi}{2} \rightarrow a = \frac{k\pi}{4} + \frac{3\pi}{16} \end{cases} \xrightarrow{x\in[0,\pi]}$$

$$\boxed{\text{بزرگترین - کمترین} = \frac{15\pi}{16}}$$

۱ $\tan(۳x)\cdot\tan(x) = 1 \longrightarrow \dfrac{\sin ۳x}{\cos ۳x} = \dfrac{\cos x}{\sin x} \longrightarrow$ حل

$\cos ۳x\cdot\cos x - \sin ۳x \sin x = ۰ \rightarrow \cos ۴x = ۰$

$۴x = ۲k\pi \pm \dfrac{\pi}{۲} \rightarrow x = \dfrac{k\pi}{۲} \pm \dfrac{\pi}{۸} \xrightarrow{x\in[\pi,۲\pi]} x = \left\{ \dfrac{۹\pi}{۸}, \dfrac{۱۳\pi}{۸}, \dfrac{۱۱\pi}{۸}, \dfrac{۱۵\pi}{۸} \right\}$

$\boxed{S = ۶\pi}$