۱۸٫۵

B - خوارزمی حضر — خورشید احمدی

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۱) ۲

$$8\cos x - \tan^2 x = 1 \longrightarrow 8\cos x = 1 + \tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x}$$

$$\xrightarrow{\cos x \neq 0} \cos^3 x = \frac{1}{8} \longrightarrow \cos x = \frac{1}{2} \longrightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{x \in [0, 2\pi]}$$

$$x : \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\} \Longrightarrow$$ ۲ جواب دارد

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۲) ۲

$$\underbrace{5\sin^2 x}_{\text{همیشه مثبت یا صفر}} + 2\cos 3x = -2 \Rightarrow 2\cos 3x \leq -2 \rightarrow \cos 3x \leq -1 \xrightarrow{-1 \leq \cos x \leq 1} \cos 3x = -1$$

$$\xrightarrow{\cos 3x = -1} \sin x = 0$$

$$\Rightarrow x = \{k\pi\} \cap \left\{ \frac{2k\pi \pm \pi}{3} \right\} \xrightarrow{x \in [-\pi, \pi]} x = \{-\pi, \pi\}$$

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۳) ۱٫۵

$$2\sin x \underbrace{\cos 2x}_{-2\sin^2 x + 1} + \sin x = 1 \rightarrow \sin x(2 - 4\sin^2 x + 1) = 1$$

$$(3 - 4\sin^2 x)$$

$$\longrightarrow 4\sin^3 x - 3\sin x - 1 = 0 \xrightarrow[ax^3 + bx^2 + cx + d]{a+b+c+d=0} \sin x = 1$$ یکی از ریشه‌هاست

$$\Rightarrow (\sin x - 1)(4\sin^2 x + 4\sin x + 1) = 0 \rightarrow (\sin x - 1)(2\sin x + 1)^2 = 0$$

$$\Rightarrow \begin{cases} \sin x = -\frac{1}{2} \longrightarrow 2k\pi + \frac{7\pi}{6}, \ 2k\pi - \frac{\pi}{6} \\ \sin x = 1 \longrightarrow 2k\pi + \frac{\pi}{2} \end{cases} \xrightarrow{x \in [0, 2\pi]} x = \left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$\boxed{S = \frac{7\pi}{2}}$$

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۴) ۲

$$\frac{1}{\sin\left(\frac{\pi}{2} + 2x\right)} + \frac{1}{\cos\left(\frac{\pi}{2} + 4x\right)} = 0 \longrightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 4x} = 0$$

$$\longrightarrow \frac{2\sin 2x - 1}{\sin 4x} = 0 \xrightarrow{\sin 4x \neq 0} \sin 2x = \frac{1}{2} \rightarrow 2x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \frac{5\pi}{6} \end{cases}$$

$$\Rightarrow x = \begin{cases} k\pi + \frac{\pi}{12} \\ k\pi + \frac{5\pi}{12} \end{cases} \xrightarrow{\sin 4x \neq 0} \begin{cases} x_1 = \frac{\pi}{12} \\ x_2 = \frac{5\pi}{12} \end{cases} \Rightarrow \overbrace{x_2 - x_1}^{\alpha} = \frac{\pi}{3} \longrightarrow \tan 2\alpha = \tan \frac{2\pi}{3} = \boxed{-\sqrt{3}}$$

(۵ $m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6} \rightarrow -\sqrt{2}\cdot m\left(\sin\left(x+\frac{\pi}{4}\right)\right) - 3\sqrt{6}\sin 2x = \sqrt{6}$

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$\rightarrow m\left(\sin\left(x+\frac{\pi}{4}\right)\right) + 3\sqrt{3}\sin 2x = -\sqrt{3} \xrightarrow{\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}} \sin\left(\frac{\pi}{2} - x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

$\Rightarrow \cancel{2} m \frac{\cancel{\sqrt{3}}}{2} + (-3)\cancel{\sqrt{3}}\sin 2x = \cancel{2}\cancel{\sqrt{3}} \rightarrow \frac{1}{3}m - 3\sin 2x = 1 \Rightarrow$

$\frac{m-3}{9} = \sin 2x \xrightarrow{\cos\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}} \frac{m-3}{9} = -\frac{1}{2} \Rightarrow \boxed{m = -\frac{3}{2}}$

$\Rightarrow x = \begin{cases} 2k\pi - \frac{\pi}{12} \\ 2k\pi - \frac{5\pi}{12} \end{cases}$

(۶ $\sin\left(x+\frac{\pi}{4}\right)\cdot\cos\left(x-\frac{\pi}{3}\right) = \sin\left(\pi-\left(x+\frac{\pi}{6}\right)\right)\cdot\sin\left(\frac{\pi}{2}-\left(x-\frac{\pi}{3}\right)\right)$

$\Rightarrow \sin\left(\frac{5\pi}{6}-x\right)\cdot\sin\left(\frac{5\pi}{6}-x\right) = 1 \Rightarrow \sin\left(\frac{5\pi}{6}-x\right) = \pm 1 \rightarrow$ ۲

$\begin{cases} \sin\left(\frac{5\pi}{6}-x\right) = \sin\left(\frac{\pi}{2}\right) \rightarrow x = \frac{\pi - 6k\pi}{3} \\ \sin\left(\frac{5\pi}{6}-x\right) = \sin\left(\frac{3\pi}{2}\right) \rightarrow x = \frac{-2\pi - 6k\pi}{3} \end{cases} \xrightarrow{x\in[0,2\pi]} \boxed{x = \left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}}$

(۷ $\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2} \rightarrow$

$\cos\left(x-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right) \Rightarrow x - \frac{\pi}{6} = 2k\pi \pm \frac{\pi}{4} \Rightarrow x = \begin{cases} \frac{24k\pi + 5\pi}{12} \\ \frac{24k\pi - \pi}{12} \end{cases}$ ۲

$\xrightarrow{x\in[-\pi,2\pi]} x = \left\{-\frac{\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12}\right\} \rightarrow \boxed{S = \frac{9\pi}{4}}$

(۸ $4\sin x\sin\left(\frac{3\pi}{2}-x\right) = -4\sin x\cos x = -2\sin 2x = 1$

$\rightarrow \sin 2x = -\frac{1}{2} \rightarrow 2x = \begin{cases} 2k\pi - \frac{\pi}{6} \rightarrow x = \frac{12k\pi - \pi}{12} \\ 2k\pi + \frac{7\pi}{6} \rightarrow x = \frac{12k\pi + 7\pi}{12} \end{cases}$ ۲ $\xrightarrow{x\in[0,2\pi]} x = \left\{\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}\right\}$

$\boxed{S = 5\pi}$

(۹ $(1+\underbrace{\cos 2a}_{2\cos^2 a})(1+\underbrace{\cos 4a}_{2\cos^2 2a})(1+\underbrace{\cos 8a}_{2\cos^2 4a}) = \frac{1}{8} \xrightarrow{\times \sin^2 a} \frac{1}{8}\sin^2 8a = \frac{1}{8}$

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$\rightarrow \sin^2 8a = 1 \rightarrow \sin 8a = \pm 1 \rightarrow \begin{cases} 8a = 2k\pi + \frac{\pi}{2} \rightarrow a = \frac{k\pi}{4} + \frac{\pi}{16} \\ 8a = 2k\pi + \frac{3\pi}{2} \rightarrow a = \frac{k\pi}{4} + \frac{3\pi}{16} \end{cases} \xrightarrow{x\in[0,\pi]}$

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$\boxed{\text{بزرگتر - صفر} = \frac{15\pi}{16}}$

حل

$\tan(3x)\cdot\tan(x) = 1 \longrightarrow \dfrac{\sin 3x}{\cos 3x} = \dfrac{\cos x}{\sin x} \longrightarrow$

$\cos 3x\cdot\cos x - \sin 3x\,\sin x = 0 \longrightarrow \cos 4x = 0$

$4x = 2k\pi \pm \dfrac{\pi}{2} \longrightarrow x = \dfrac{k\pi}{2} \pm \dfrac{\pi}{8} \xrightarrow{x\in[\pi,2\pi]} x = \left\{\dfrac{9\pi}{8}, \dfrac{13\pi}{8}, \dfrac{11\pi}{8}, \dfrac{15\pi}{8}\right\}$

$\boxed{S = 6\pi}$

9

$1+\cos 2* = 2\cos^2 *$

$\left.\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases}\right\} \rightarrow 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \dfrac{1}{8}$

$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \dfrac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \pm\dfrac{1}{8} \xrightarrow[\sin\alpha\neq 0]{\times \sin\alpha\ *}$

$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\dfrac{\sin\alpha}{8} \longrightarrow \dfrac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\dfrac{\sin\alpha}{8}$

$\dfrac{1}{2}\times\dfrac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \pm\dfrac{\sin\alpha}{8} \longrightarrow \dfrac{1}{\cancel{8}}\sin 8\alpha = \pm\dfrac{\sin\alpha}{\cancel{8}}$

$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا}\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا}\ 2k\pi + \pi + \alpha \end{cases}$

$\alpha = \dfrac{2k\pi}{7} \xrightarrow{\max A} k=3 \rightarrow x = \dfrac{6\pi}{7}$

$\alpha = \dfrac{2k\pi}{9} \xrightarrow{\max A} k=4 \rightarrow x = \dfrac{8\pi}{9}$

$\alpha = \dfrac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \rightarrow x = \dfrac{5\pi}{7}$

$\alpha = \dfrac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \rightarrow x = \dfrac{7\pi}{9}$

$\rightarrow \boxed{\max A = \dfrac{8\pi}{9}}$

* A نمی‌تواند برابر خود π باشد چون طبق * $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!

۳

$$\cos 2x = 1 - 2\sin^2 x \quad (\text{فرمول طلایی})$$

۳ ٤ ۳

$$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$$

$$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$$

$$k=0 \longrightarrow x = \frac{\pi}{6}, \quad k=1 \longrightarrow x = \frac{5\pi}{6}, \quad k=2 \longrightarrow x = \frac{9\pi}{6}$$

$$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$$

۵

$$\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3}m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3}m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m = 6}$$