۱۸٫۵

نام و نام خانوادگی ...دلارام میرزابابائیان... پاسخنامه تشریحی تکلیف شماره ...۱۷... کلاس ...دوازدهم دختر B...

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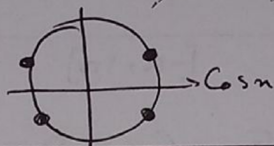
$8\cos x - \tan^2 x = 1 \quad [0, 2\pi]$ تعداد

$8\cos x - \dfrac{\overbrace{\sin^2 x}^{1-\cos^2 x}}{\cos^2 x} = 1$

$\xrightarrow{\times \cos x} \dfrac{8\cos^2 x - \overbrace{\sin^2 x}^{1-\cos^2 x}}{\cos^2 x} = 1 \longrightarrow 8\cos^2 x - 1 + \cancel{\cos^2 x} = \cancel{\cos^2 x} \longrightarrow 8\cos^2 x = 1$

$\cos^2 x = \dfrac{1}{8}$

$\cos x = \pm \dfrac{1}{2\sqrt{2}}$



از 0 تا 2π ← ۴ جواب

دقت !

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$5\sin^2(x) + 2\cos(3x) = -2 \quad [-\pi, \pi]$ تعداد

$\sin^2 x = 0 \longrightarrow x = k\pi \quad (1)$

$\cos 3x = -1 \longrightarrow 3x = 2k\pi + \pi \rightarrow x = \dfrac{2k\pi}{3} + \dfrac{\pi}{3} \quad (2)$

$\xrightarrow{(1) \cap (2)} \pi, -\pi$

۲ جواب

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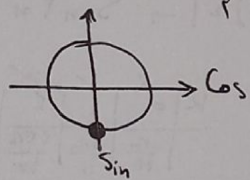
$2\sin(x)\underbrace{\cos(2x)}_{1-2\sin^2(x)} + \sin(x) = 1 \longrightarrow 2\sin(x) - 4\sin^3(x) + \sin(x) = 1 \quad [0, 2\pi]$

مجموع $= \dfrac{3\pi}{2}$

$3\sin x - 4\sin^3(x) - 1 = 0 \longrightarrow \sin x = -1$



۱ جواب

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$\dfrac{1}{\sin\left(\frac{\pi+4x}{2}\right)} + \dfrac{1}{\cos\left(\frac{\pi+8x}{2}\right)} = 0 \quad [0, \pi]$

$\dfrac{1}{\cos 2x} + \dfrac{1}{-\sin 4x} = 0 \longrightarrow \sin 4x = \cos 2x \longrightarrow 2\sin 2x \cos 2x = \cos 2x$

$\cos 2x = 0$ غ‌ق‌ق (در دامنه نیست)

$\sin 2x = \frac{1}{2} \rightarrow 2x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

$\dfrac{5\pi}{12} - \dfrac{\pi}{12} = \dfrac{4\pi}{12} = \dfrac{\pi}{3} = a \qquad \boxed{\tan(\underbrace{2a}_{120^\circ}) = -\sqrt{3}}$

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$\cos\left(x + \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}}$

$\dfrac{\sqrt{2}}{2}(\cos x - \sin x) = \dfrac{1}{\sqrt{3}}$

$\cos x - \sin x = \dfrac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{توان } 2} \cancel{\cos^2 x + \sin^2 x} - 2\cos x \sin x = \dfrac{2}{3}$

$2\cos x \sin x = \dfrac{1}{3}$

$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$

$m\left(\dfrac{\sqrt{2}}{\sqrt{3}}\right) - \cancel{3}\sqrt{6} \times \dfrac{1}{\cancel{3}} = \sqrt{6} \longrightarrow m = 2\sqrt{6} \times \dfrac{\sqrt{3}}{\sqrt{2}} = \boxed{6}$

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$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{3})=1 \qquad [0,2\pi]$

$\sin(x+\frac{\pi}{6})\sin(\frac{\pi}{2}+x-\frac{\pi}{3})=1 \to \sin(x+\frac{\pi}{6})\sin(x+\frac{\pi}{6})=1$

$\sin^2(x+\frac{\pi}{6})=1 \to x+\frac{\pi}{6}=k\pi+\frac{\pi}{2} \to x=k\pi+\frac{\pi}{3} \to \frac{3k\pi+\pi}{3}$ 6

K	0 ✓	1 ✓	2 ✗
	$\frac{\pi}{3}$	$\frac{4\pi}{3}$	$\frac{7\pi}{3}$

۲ جواب

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$\sin x+\sqrt{3}\cos x=\sqrt{2} \qquad [-\pi,2\pi]$

$\xrightarrow{\div 2} \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{2}}{2} \to \cos\frac{\pi}{3}\sin x+\sin\frac{\pi}{3}\cos x=\sin\frac{\pi}{4}$

$\xrightarrow[\sin]{\text{بسط}} \sin(\frac{\pi}{3}+x)=\sin\frac{\pi}{4}$ ۱,۵ 7

$\frac{4\pi}{12}+x=2k\pi+\cancel{\frac{\pi}{4}}\,\frac{3\pi}{12} \to x=2k\pi-\frac{\pi}{12}$ ① $\frac{24k\pi+\pi}{12}$

①

K	-1 ✗	0 ✓	1 ✗
	$-\frac{23\pi}{12}$	$\frac{\pi}{12}$	$\frac{25\pi}{12}$

$\cancel{\frac{\pi}{3}}\,\frac{4\pi}{12}+x=2k\pi+\underbrace{\pi-\cancel{\frac{\pi}{4}}}_{\frac{3\pi}{4}\ \frac{9\pi}{12}} \to x=2k\pi+\frac{5\pi}{12}$ ② $\frac{24k\pi+5\pi}{12}$

②

K	-1 ✗	0 ✓	1 ✗
	$-\frac{19\pi}{12}$	$\frac{5\pi}{12}$	$\frac{29\pi}{12}$

$\frac{\pi}{12}+\frac{5\pi}{12}=\frac{6\pi}{12}=\boxed{\frac{\pi}{2}}$

$4\sin x\underbrace{\sin(\frac{3\pi}{2}-x)}_{-\cos x}=1 \qquad [0,2\pi]$

$\frac{7\pi+19\pi+11\pi+23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$

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$-4\sin x\cos x=1$ 8

$-2(\underbrace{2\sin x\cos x}_{\sin 2x})=1 \to \sin 2x=-\frac{1}{2}$; $\sin\frac{7\pi}{6}$

$\to 2x=2k\pi+\frac{7\pi}{6} \to x=k\pi+\frac{7\pi}{12}$; $\frac{12k\pi+7\pi}{12}$

$2x=2k\pi+\underbrace{\pi-\frac{7\pi}{6}}_{-\frac{\pi}{6}} \to x=k\pi-\frac{\pi}{12}$; $\frac{12k\pi-\pi}{12}$

K	0	1 ✓	2 ✗
	$\frac{7\pi}{12}$	$\frac{19\pi}{12}$	$\frac{31\pi}{12}$

K	0 ✗	1 ✓	2 ✓	3 ✗
	$-\frac{\pi}{12}$	$\frac{11\pi}{12}$	$\frac{23\pi}{12}$	$\frac{35\pi}{12}$

$\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{2\cos^2 4\alpha}=\frac{1}{8}$

$Max \to \frac{8\pi}{9}$ ۲

$\sin 2\alpha \quad \sin 4\alpha \quad \sin 8\alpha$

$\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha=\pm\frac{1}{8} \xrightarrow[\alpha\neq\pi]{\times\sin\alpha\neq 0} 8\sin\alpha\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha=\pm\sin\alpha$ 9

$* \ 2\times\cancel{4}\times 2\sin\alpha\cos\alpha$

$\sin 8\alpha=\pm\sin\alpha \qquad 0,\frac{2\pi}{7},\frac{4\pi}{7},\frac{6\pi}{7} \qquad 0,\frac{2\pi}{9},\frac{4\pi}{9},\frac{6\pi}{9},\frac{8\pi}{9}$

$\sin 8\alpha=\sin\alpha \to 8\alpha=2k\pi+\alpha \to \alpha=\frac{2k\pi}{7}$

$8\alpha=2k\pi+\pi-\alpha \to \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$

$\frac{\pi}{9},\frac{3\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\pi$ * غ.ق.ق

$\sin 8\alpha=-\sin\alpha \to 8\alpha=2k\pi-\alpha \to \alpha=\frac{2k\pi}{9}$

$8\alpha=2k\pi+\pi+\alpha \to \alpha=\frac{2k\pi+\pi}{7}$

$\frac{\pi}{7},\frac{3\pi}{7},\frac{5\pi}{7},$

$\tan(3x)\tan(x)=1 \qquad [\pi,2\pi]$

$\to \tan(3x)=\cot(x) \to \tan(\frac{\pi}{2}-x)$

$3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi+\frac{\pi}{2} \to x=\frac{k\pi}{4}+\frac{\pi}{8}\ \boxed{\frac{2k\pi+\pi}{8}}$ ۲ 10

K	0 ✗	1 ✗	....	4 ✓	5 ✓	6 ✓	7 ✓	8 ✗
	$\frac{\pi}{8}$	$\frac{3\pi}{8}$		$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$	$\frac{17\pi}{8}$

$\frac{9\pi+11\pi+13\pi+15\pi}{8}=\frac{48\pi}{8}=\boxed{6\pi}$

(۱)

$$8\cos x = 1+\tan^2 x \rightarrow 8\cos x = \frac{1}{\cos^2 x} \rightarrow 8\cos^3 x = 1$$

$$\cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{0\le x\le 2\pi} \boxed{x = \frac{\pi}{3} \text{ یا } \frac{5\pi}{3}}$$

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$\cos 2x = 1-2\sin^2 x$ (فرمول طلایی)

۳ ٤ ۳ ت

$$2\sin x(1-2\sin^2 x)+\sin x = 1 \rightarrow 3\sin x - 4\sin^3 x = 1$$

$$\sin 3x = 1 \rightarrow 3x = 2k\pi+\frac{\pi}{2} \rightarrow x = \frac{2k\pi}{3}+\frac{\pi}{6} \xrightarrow{0\le x\le 2\pi}$$

$$k=0 \rightarrow x=\frac{\pi}{6}, \quad k=1 \rightarrow x=\frac{5\pi}{6}, \quad k=2 \rightarrow x=\frac{9\pi}{6}$$

$$S = \frac{\pi}{6}+\frac{5\pi}{6}+\frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$$

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(۷) عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \rightarrow \sin\left(x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi-\frac{\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0,1 \rightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x+\frac{\pi}{3} = 2k\pi+\frac{3\pi}{4} \rightarrow x = 2k\pi+\frac{5\pi}{12} \xrightarrow{-\pi\le x\le 2\pi} k=0 \rightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12}-\frac{\pi}{12}+\frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$