

سوال ۱) $\lambda \cos x - \tan^2 x = 1 \rightarrow \lambda \cos x = 1 + \frac{\sin^2 x}{\cos^2 x} \rightarrow \lambda \cos x = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \rightarrow \cos^2 x = \frac{1}{\lambda} \rightarrow \cos x = \frac{1}{\sqrt{\lambda}} \xrightarrow{[0, 2\pi]} x = \frac{\pi}{\sqrt{\lambda}}, \frac{5\pi}{\sqrt{\lambda}}$ جواب دارد

سوال ۲) $5 \sin^2 x + 2 \cos^4 x = -2 \rightarrow 5(1 - \cos^2 x) + 2 \cos^4 x = -2 \rightarrow \lambda \cos^2 x - 5 \cos^4 x - 4 \cos^2 x + 7 = 0 \xrightarrow{\cos x = t} \lambda t^2 - 5t^4 - 4t^2 + 7 = 0$

$t = -1 \rightarrow \lambda(-1) - 5(1) - 4(-1) + 7 = 0 \rightarrow \cos x = -1 \xrightarrow{[-\pi, \pi]} x = \pi, -\pi$

$\lambda t^2 - 5t^4 - 4t^2 + 7 \mid \frac{t+1}{\lambda t^2 - 13t + 7} \xrightarrow{\Delta < 0}$ زیرا در بازه $[-\pi, \pi]$ تنها جواب دارد
 $\lambda t^2 - 5t^4 - 4t^2 + 7$
 $-13t^2 - 13t + 7$
 $\frac{7t + 7}{7t + 7}$
 $\frac{0}{0}$

سوال ۳) $\sqrt{3} \sin x \cos^2 x + \sin x = 1 \rightarrow \sqrt{3} \sin x (1 - \sin^2 x) + \sin x = 1 \rightarrow \sqrt{3} \sin x - \sqrt{3} \sin^3 x + \sin x = 1 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1 \rightarrow x = \frac{\pi}{2} + k\pi, \frac{3\pi}{2} + k\pi$

$\xrightarrow{[0, 2\pi]} x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ جمع جواب ها: $\frac{\pi}{2} + \frac{5\pi}{2} + \frac{3\pi}{2} = \frac{9\pi}{2} = \frac{5\pi}{2}$

سوال ۴) $\frac{1}{\sin(\frac{\pi}{\sqrt{2}} + 2x)} + \frac{1}{\cos(\frac{\pi}{\sqrt{2}} + 2x)} = 0 \rightarrow \frac{1}{\cos 2x} - \frac{1}{\sin 2x} = 0 \rightarrow \cos 2x = \sin 2x \rightarrow \sin(\frac{\pi}{\sqrt{2}} + 2x) = \sin 2x \rightarrow 2x = 2k\pi + \frac{\pi}{\sqrt{2}} + 2n$
 $\rightarrow 2x = 2k\pi + \frac{\pi}{\sqrt{2}} \rightarrow x = k\pi + \frac{\pi}{2\sqrt{2}} \xrightarrow{[0, 2\pi]} x = \frac{\pi}{2\sqrt{2}} \rightarrow$ غیر قابل تبدیل زیرا خارج باید صفر شود

$f(x) = 2k\pi + \pi - \frac{\pi}{\sqrt{2}} - 2x \rightarrow 2x = 2k\pi + \frac{\pi}{\sqrt{2}} \rightarrow x = \frac{k\pi}{\sqrt{2}} + \frac{\pi}{2\sqrt{2}} \xrightarrow{[0, 2\pi]} x = \frac{\pi}{2\sqrt{2}}, \frac{5\pi}{2\sqrt{2}}$ قابل تبدیل $\Rightarrow \alpha = \frac{5\pi}{2\sqrt{2}} - \frac{\pi}{2\sqrt{2}} = \frac{2\pi}{\sqrt{2}} \rightarrow \tan \alpha = \tan \frac{2\pi}{\sqrt{2}} = -\sqrt{2}$

سوال ۵) $m(\cos x - \sin x) - 3\sqrt{4} \sin 2x = \sqrt{4} \rightarrow \cos(x + \frac{\pi}{4}) = \frac{\sqrt{2}}{\sqrt{2}} \xrightarrow{\text{تبدیل}} \cos^2(x + \frac{\pi}{4}) = \frac{1 + \cos(2x + \frac{\pi}{2})}{2} = \frac{1}{2} \rightarrow 1 - \sin 2x = \frac{1}{2} \rightarrow \sin 2x = \frac{1}{2}$

$m(\cos x - \sin x) - 3\sqrt{4} \times \frac{1}{\sqrt{2}} = \sqrt{4} \rightarrow m \times \sqrt{2} \times \sin(\frac{\pi}{4} - x) = 2\sqrt{4} \rightarrow m \times \sqrt{2} \times \cos(x + \frac{\pi}{4}) = 2\sqrt{4} \rightarrow m \times \sqrt{2} \times \frac{1}{\sqrt{2}} = 2\sqrt{4} \rightarrow m = 4$

سوال ۶) $\sin(x + \frac{\pi}{4}) \cos(x - \frac{\pi}{4}) = 1 \rightarrow \sin(x + \frac{\pi}{4}) \cos(\frac{\pi}{4} - x) = 1 \xrightarrow{\text{تبدیل}} \sin^2(x + \frac{\pi}{4}) = 1 \rightarrow \sin(x + \frac{\pi}{4}) = \pm 1 \rightarrow x + \frac{\pi}{4} = k\pi + \frac{\pi}{2}$

$\rightarrow x = k\pi + \frac{\pi}{4} \xrightarrow{[0, 2\pi]} x = \frac{\pi}{4}, \frac{5\pi}{4}$ عدد در این بازه ۲ جواب دارد

سوال ۷) $\sin x + \sqrt{3} \cos x = \sqrt{2} \rightarrow \frac{a}{|a|} \sqrt{a^2 + b^2} \sin(x + \alpha) \rightarrow \tan \alpha = \frac{b}{a} \rightarrow \tan \alpha = \sqrt{3} \rightarrow \alpha = \frac{\pi}{3} \rightarrow \sqrt{2} \sin(x + \frac{\pi}{3}) = \sqrt{2} \rightarrow \sin(x + \frac{\pi}{3}) = \frac{\sqrt{2}}{\sqrt{2}}$

$\sin(x + \frac{\pi}{3}) = \sin \frac{\pi}{4} \rightarrow x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{[-\pi, 2\pi]} x = -\frac{\pi}{12}, \frac{23\pi}{12}$
 $\downarrow$
 $x + \frac{\pi}{3} = 2k\pi + \pi - \frac{\pi}{4} \rightarrow x = 2k\pi + \frac{11\pi}{12} \xrightarrow{[-\pi, 2\pi]} x = \frac{5\pi}{12}, \frac{17\pi}{12}$
 $\left. \begin{array}{l} \frac{23\pi}{12} + \frac{5\pi}{12} - \frac{\pi}{3} = \frac{23\pi}{12} + \frac{5\pi}{12} - \frac{4\pi}{12} = \frac{24\pi}{12} = 2\pi \\ \frac{17\pi}{12} + \frac{11\pi}{12} - \frac{\pi}{3} = \frac{17\pi}{12} + \frac{11\pi}{12} - \frac{4\pi}{12} = \frac{24\pi}{12} = 2\pi \end{array} \right\} \rightarrow \frac{23\pi}{12} + \frac{11\pi}{12} + \frac{17\pi}{12} + \frac{5\pi}{12} = \frac{56\pi}{12} = \frac{14\pi}{3}$

سوال ۸) $\sqrt{3} \sin x \sin(\frac{3\pi}{4} - x) = 1 \rightarrow \sqrt{3} \sin x \cos x = -1 \rightarrow \sin 2x = -\frac{1}{\sqrt{3}} \rightarrow 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \xrightarrow{[0, 2\pi]} x = \frac{7\pi}{12}, \frac{19\pi}{12}$
 $\downarrow$
 $2x = 2k\pi + \frac{11\pi}{6} \rightarrow x = k\pi + \frac{11\pi}{12} \xrightarrow{[0, 2\pi]} x = \frac{11\pi}{12}, \frac{23\pi}{12}$
 $\left. \begin{array}{l} \frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi \\ \frac{19\pi}{12} + \frac{11\pi}{12} + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi \end{array} \right\} \rightarrow \frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} + \frac{23\pi}{12} = \frac{60\pi}{12} = 5\pi$

(سوال 9)

$$(1 + \cos^p \alpha)(1 + \cos^q \alpha)(1 + \cos^r \alpha) = \frac{1}{\lambda} \rightarrow (\sqrt[p]{\cos^p \alpha})(\sqrt[q]{\cos^q \alpha})(\sqrt[r]{\cos^r \alpha}) = \frac{1}{\lambda} \times \frac{\sin^p \alpha}{\sin^p \alpha}$$

$$\underbrace{\sin^p \alpha (\sqrt[p]{\cos^p \alpha})}_{\frac{1}{\sqrt[p]{\sin^p \alpha}} \sin^p \alpha} \underbrace{(\sqrt[q]{\cos^q \alpha})}_{\frac{1}{\sqrt[q]{\sin^q \alpha}} \sin^q \alpha} \underbrace{(\sqrt[r]{\cos^r \alpha})}_{\frac{1}{\sqrt[r]{\sin^r \alpha}} \sin^r \alpha} \times \frac{1}{\sin^p \alpha} = \frac{1}{\lambda} \rightarrow \sin^p \alpha = \sin^p \lambda \alpha \rightarrow \sin \lambda \alpha = \sin \alpha \rightarrow \lambda \alpha = \sqrt[p]{k} \pi + \alpha \rightarrow \alpha = \frac{\sqrt[p]{k} \pi}{\sqrt[p]{\lambda}}$$

$$\lambda \alpha = \sqrt[p]{k} \pi + \pi - \alpha \rightarrow \alpha = \frac{\sqrt[p]{k} \pi}{\sqrt[p]{\lambda}} + \frac{\pi}{\sqrt[p]{\lambda}}$$

$$\sin \lambda \alpha = \sin(-\alpha) \rightarrow \lambda \alpha = \sqrt[p]{k} \pi - \alpha \rightarrow \alpha = \frac{\sqrt[p]{k} \pi}{\sqrt[p]{\lambda}}$$

$$\lambda \alpha = \sqrt[p]{k} \pi + \pi + \alpha \rightarrow \alpha = \frac{\sqrt[p]{k} \pi}{\sqrt[p]{\lambda}} + \frac{\pi}{\sqrt[p]{\lambda}}$$

$$\alpha = \pi \leftarrow k = \sqrt[p]{\lambda} \leftarrow \text{مطلوب}$$

(سوال 10)

$$\tan^p x \tan^q x = 1 \rightarrow \tan^p x = \frac{1}{\tan^q x} \rightarrow \tan^p x = \cot^q x \rightarrow \tan^p x = \tan\left(\frac{\pi}{p} - x\right) \rightarrow \sqrt[p]{p} x = k\pi + \frac{\pi}{\sqrt[p]{p}} - x$$

$$\sqrt[p]{p} x = k\pi + \frac{\pi}{\sqrt[p]{p}} \rightarrow x = \frac{k\pi}{\sqrt[p]{p}} + \frac{\pi}{\sqrt[p]{\lambda}} \xrightarrow{[0, 2\pi]} x = \frac{9\pi}{\sqrt[p]{\lambda}}, \frac{11\pi}{\sqrt[p]{\lambda}}, \frac{13\pi}{\sqrt[p]{\lambda}}, \frac{15\pi}{\sqrt[p]{\lambda}} \xrightarrow{\text{جمع}} \frac{9\pi}{\sqrt[p]{\lambda}} + \frac{11\pi}{\sqrt[p]{\lambda}} + \frac{13\pi}{\sqrt[p]{\lambda}} + \frac{15\pi}{\sqrt[p]{\lambda}} = 4\pi$$