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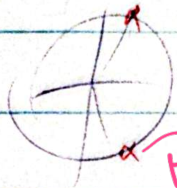
نیلو حیدری — بسم الله الرحمن الرحیم — کلاس سه‌شنبه‌ها

سوال ۱

$$8\cos x = 1 + \tan^2 \rightarrow 8\cos x = \frac{1}{\cos^2}$$

دو جواب

$$8\cos^3 x = 1 \rightarrow \cos x = \frac{1}{2}$$



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سوال ۳

$$2\sin\alpha \cos 2\alpha + \sin\alpha = 1$$

$$\sin\alpha(2\cos 2\alpha + 1) = 1 \xrightarrow{\sin^2\alpha = \frac{1-\cos 2\alpha}{2}} \sin\alpha(2(1-2\sin^2\alpha)+1) =$$

$$\sin\alpha(2 - 4\sin^2\alpha + 1) \rightarrow \sin\alpha(3 - 4\sin^2\alpha) \rightarrow$$

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$$3\sin\alpha - 4\sin^3\alpha = \sin 3\alpha = 1$$

$$3\alpha = 2k\pi + \frac{\pi}{2} \rightarrow \alpha = \frac{4k\pi}{6} + \frac{\pi}{6} \rightarrow \frac{4k\pi + \pi}{6} \begin{cases} \frac{\pi}{6} \\ \rightarrow \frac{5\pi}{6} \\ \rightarrow \frac{9\pi}{6} \end{cases}$$

$$S \rightarrow \frac{15\pi}{6} = \frac{5\pi}{2} = \boxed{2/5\pi}$$

سوال ۴

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} \rightarrow \frac{1}{\cos 2x} + \frac{1}{-\sin 4x}$$

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$$-\sin 4x + \cos 2x = 0 \rightarrow \cos 2x = \sin 4x \rightarrow \sin 4x = \sin(\frac{\pi}{2} - 2x)$$

$$4x = 2k\pi + \frac{\pi}{2} - 2x \rightarrow 6x = 2k\pi + \frac{\pi}{2} \rightarrow \frac{\pi}{12}$$

$$4x = 2k\pi + \frac{\pi}{2} + 2x \rightarrow 2x = 2k\pi + \frac{\pi}{4} \rightarrow \frac{\pi}{4}$$

$$\frac{\pi}{4} - \frac{\pi}{12} = \frac{\pi}{6} \quad \tan 2x \rightarrow \tan\frac{2\pi}{6} \rightarrow \boxed{\sqrt{3}}$$

سوال ۲

$5\sin^2 x + 2\cos(3x) = -2 \longrightarrow$ تعداد جواب‌ها

$\sin^2 = 0$ و $2\cos 3x = -2$

$\sin^2 = 0 \Rightarrow \sin x = 0$

$x_1 = k\pi$, $2\cos 3x = -2 \longrightarrow \cos 3x = -1 \longrightarrow 3x = 2k\pi + \pi$

$x_2 = \frac{2k\pi}{3} + \frac{\pi}{3}$

$x_1 = -\pi, 0, \pi$

$x_2 = \frac{\pi}{3}, \pi, -\frac{\pi}{3}, -\pi$

$\Rightarrow \cap = \pi, -\pi \longrightarrow 2$

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دو جواب

Farzin ... Book
Subject:
Date:

سوال ۵

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$\cos x \cos\frac{\pi}{4} - \sin x \sin\frac{\pi}{4} = \frac{1}{\sqrt{3}}$

$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \longrightarrow \cos x - \sin x = \frac{\sqrt{6}}{3}$

$m\left(\frac{\sqrt{6}}{3}\right) - 3\sqrt{6}\sin 2x = \sqrt{6} \longrightarrow m - 9\sin 2x = 3$

$m = 3 + 9\sin 2x \overset{\frac{4}{9}}{} \Rightarrow m = 3 + 4 \longrightarrow \boxed{m = 7}$

$(\cos x - \sin x)^2 \longrightarrow \cos^2 x + \sin^2 x - 2\sin x\cos x = \frac{6}{9} \longrightarrow \frac{4}{9} = 2\sin x\cos x$

سوال ۶

$\sin(x+\frac{\pi}{4})\cos(\frac{\pi}{4}-x) = 1$

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$2\sin(x+\frac{\pi}{4}) = 1 \longrightarrow \sin(x+\frac{\pi}{4}) = \frac{1}{2} \longrightarrow x+\cancel{\frac{\pi}{4}} = 2k\pi + \cancel{\frac{\pi}{4}}$

$\boxed{x = 0, \pi}$

$x + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4} \longrightarrow x = 2k\pi + \frac{4\pi}{4} \longrightarrow \boxed{x = \frac{4\pi}{4}}$

سوال ۷

$2\sin(x + \frac{\pi}{3}) = \sqrt{2}$

$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} \longrightarrow x + \frac{4\pi}{12} = 2k\pi + \frac{3\pi}{12} \longrightarrow x = 2k\pi - \frac{\pi}{12}$

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$x + \frac{4\pi}{12} = 2k\pi + \frac{9\pi}{12} \longrightarrow x = 2k\pi + \frac{5\pi}{12}$

$\frac{\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}$

سوال ۸

$-4\sin\cos(x) = 1 \to 2\sin 2x = -1$

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$\sin 2x = -\frac{1}{2} \to 2x = 2k\pi + \frac{2\pi}{3} \to x = k\pi + \frac{\pi}{3}$

$2x = 2k\pi + \frac{4\pi}{3} \to x \to k\pi + \frac{2\pi}{3}$

$\frac{\pi}{3}, \pi + \frac{\pi}{3}, \frac{2\pi}{3}, \pi + \frac{2\pi}{3} \to 4\pi$

سوال ۹

$(1+\cos\alpha)(1+\cos 2\alpha)(1+\cos 8\alpha) = \frac{1}{8}$

$2\cos^2\frac{\alpha}{2} \quad 2\cos^2\alpha \quad 2\cos^2 4\alpha$

$\frac{1}{4}\sin^2\alpha \quad \frac{1}{4}\sin^2 2\alpha \quad \frac{1}{4}\sin^2 8\alpha$

$8\,\frac{\sin^2\frac{\alpha}{2}\cdot\cos^2\frac{\alpha}{2}\cdot\cos^2 2\alpha\cdot\cos^2 4\alpha}{\sin^2\frac{\alpha}{2}} = \frac{\sin^2 8\alpha}{\sin^2\alpha} = 1$

$\sin 8\alpha = \sin\alpha \to 8\alpha = 2k\pi + \alpha \to \alpha = \frac{2k\pi}{7}$

$8\alpha = 2k\pi + \pi - \alpha \to \alpha = \frac{2k\pi + \pi}{9}$

$\sin 8\alpha = \sin(-\alpha) \to 8\alpha = 2k\pi - \alpha \to \alpha = \frac{2k\pi}{9}$

$8\alpha = 2k\pi + \pi + \alpha \to \alpha = \frac{2k\pi + \pi}{7}$

$\max \to \pi$

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سوال ۱۰

$\tan 2x = \frac{1}{\tan x} \to \tan 2x = \cot x$

$\tan 2x = \tan(\frac{\pi}{2} - x) \to 3x = k\pi + \frac{\pi}{2} - x$

$\to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$

$\pi + \frac{\pi}{8}, \quad \frac{5\pi}{4} + \frac{\pi}{8}, \quad \frac{4\pi}{4} + \frac{\pi}{8}, \quad \frac{13\pi}{4} + \frac{\pi}{8}$

$\frac{22\pi}{4} + \frac{4\pi}{4} \to \frac{26\pi}{4} = 4\pi$

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$$\cos\left(x+\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{3}}{3} \longrightarrow \cos x - \sin x = \frac{\frac{\sqrt{3}}{3}}{\frac{1}{\sqrt{2}}}$$

$$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{گویا}} \frac{\sqrt{6}}{3} \longrightarrow \boxed{\cos x - \sin x = \frac{\sqrt{6}}{3}}$$

$$(\cos x - \sin x)^2 = 1 - \sin 2x = \frac{6}{9} \longrightarrow \boxed{\sin 2x = \frac{1}{3}}$$

$$m(\cos x - \sin x) - 3\sqrt{6}(\sin 2x) = \sqrt{6} \longrightarrow \frac{\sqrt{6}}{3} m - 3\sqrt{6}\left(\frac{1}{3}\right) = \sqrt{6}$$

$$\frac{\sqrt{6}}{3} m - \sqrt{6} = \sqrt{6} \longrightarrow \frac{m\sqrt{6}}{3} = 2\sqrt{6} \longrightarrow \boxed{m = 6}$$

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$$\cos\left(x - \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} - x\right)$$

$$\frac{\pi}{3} - x + x + \frac{\pi}{6} = \frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3} - x\right) = \sin\left(x + \frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right) = 1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right) = \pm 1 \longrightarrow x + \frac{\pi}{6} = k\pi + \frac{\pi}{2}$$

$$x = k\pi + \frac{\pi}{3} \xrightarrow{0 \le x \le 2\pi} k=0 \to x = \frac{\pi}{3}\ ,\quad k=1,\ x = \frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

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۷ عبارت را در $\frac{1}{2}$ ضرب می‌کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0,1 \to x = -\frac{\pi}{12},\ \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k=0 \to x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

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$$\sin\left(\frac{3\pi}{2}-x\right) = -\cos x$$

$$4\sin x(-\cos x)=1 \longrightarrow -2(2\sin x\cos x)=1 \longrightarrow \sin 2x=-\frac{1}{2}$$

$$2x = 2k\pi-\frac{\pi}{6} \ \text{یا}\ 2k\pi+\frac{7\pi}{6} \longrightarrow x = \overset{2}{k\pi-\frac{\pi}{12}} \ \text{یا}\ \overset{1}{k\pi+\frac{7\pi}{12}}$$

$$0\leq x\leq 2\pi \longrightarrow \begin{cases} \overset{1}{} \ k=0,1 \longrightarrow x=\frac{7\pi}{12},\ \frac{19\pi}{12} \\ \underset{2}{} \ k=1,2 \longrightarrow x=\frac{11\pi}{12},\ \frac{23\pi}{12} \end{cases}$$

$$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

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$$1+\cos 2x = 2\cos^2 x \ \bigstar$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \overset{?}{\longrightarrow} 8\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\cos 2\alpha\cos 4\alpha = \frac{\pm 1}{8} \ \underset{\sin\alpha\neq 0}{\xrightarrow{\times\sin\alpha\ \bigstar}}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\cos 4\alpha)}_{\sin 8\alpha} = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha=\frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x=\frac{6\pi}{7}$$

$$\alpha=\frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x=\frac{8\pi}{9}$$

$$\alpha=\frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x=\frac{5\pi}{7} \qquad \longrightarrow \boxed{\max A=\frac{8\pi}{9}}$$

$$\alpha=\frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x=\frac{7\pi}{9}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!