

الف)  $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \quad \Delta = 25$

$\sin x = \frac{-3 \pm 5}{4} \rightarrow \begin{cases} -2 \text{ ن. غ.} \\ \frac{1}{2} \text{ ن. ص.} \end{cases}$    $\sin x = \sin \frac{\pi}{6} \rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \pi - \frac{\pi}{6} \end{cases}$

ب)  $\cos^2 x - \sin^2 x + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \quad \Delta = 25$

$\cos x = \frac{-1 \pm 5}{4} \rightarrow \begin{cases} 1 \\ -\frac{3}{2} \text{ ن. غ.} \end{cases}$    $x = 2k\pi$

الف)  $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin 4x \rightarrow -\cos 2x = 2\sin 2x \cos 2x$

$\rightarrow -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2} \rightarrow \sin \frac{1}{2}x = \sin(-\frac{\pi}{6}) \rightarrow \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi + \pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \end{cases}$

ب)  $\frac{2\cos^2 x - 1}{\cos 2x} + \frac{2\sin x \cos x}{\sin 2x} = 0 \rightarrow \cos 2x + \sin 2x = 0 \xrightarrow{\wedge 2} \frac{\cos^2 2x + \sin^2 2x + 2\sin 2x \cos 2x}{\sin 4x} = 0$

$\sin 4x = -1 \rightarrow 4x = 2k\pi + \frac{3\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{8}$

الف)  $\frac{\cos x}{\sin x} (2\cos x + 1) = \frac{1}{\sin x} \rightarrow \frac{2\cos^2 x + \cos x}{\sin x} = \frac{1}{\sin x} \rightarrow 2\cos^2 x + \cos x = 1$

$\rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow -1 \rightarrow x = 2k\pi + \pi$   
 $\frac{1}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$

ب)  $\sin^2 x - \sin x \cos x = 1 \rightarrow \sin x \cos x = \cos^2 x \Rightarrow \cos^2 x + \sin x \cos x = 0$

$\cos x (\cos x + \sin x) = 0 \rightarrow \cos x = 0 \rightarrow x = 2k\pi \pm \frac{\pi}{2}$

$\rightarrow \cos x + \sin x = 0 \rightarrow \cos x = -\sin x \rightarrow \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

الف)  $\cos(\frac{\pi}{4} + x) \cos(x - \frac{\pi}{4}) = \frac{1}{4} \rightarrow \cos(\frac{\pi}{4} + x) \cos(\frac{\pi}{4} - x) \xrightarrow{\frac{\pi}{4} + x + \frac{\pi}{4} - x = \frac{\pi}{2}} \cos(\frac{\pi}{4} + x) \sin(\frac{\pi}{4} + x)$

$= \frac{1}{2} \sin(\frac{\pi}{2} + 2x) = \frac{1}{2} \cos 2x = \frac{1}{4} \rightarrow \cos 2x = \frac{1}{2} \rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6}$

ب)  $\cos(2x - \frac{\pi}{9}) = -\sin 2x = \cos(\frac{\pi}{2} + 2x) \rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \rightarrow \text{جواب ندارد}$

$2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \rightarrow 4x = 2k\pi - \frac{7\pi}{18} \rightarrow x = \frac{k\pi}{2} - \frac{7\pi}{72}$

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \rightarrow \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$

$= 2\cos 2x + 1 = 1 \rightarrow 2\cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \rightarrow x = k\pi \pm \frac{\pi}{4}$

$$\frac{\sin \frac{\pi}{2}}{1 + \cos \frac{\pi}{2}} = \frac{1}{\sin \frac{\pi}{2}} + \frac{\cos \frac{\pi}{2}}{\sin \frac{\pi}{2}} \rightarrow \sin^2 \frac{\pi}{2} = \cos^2 \frac{\pi}{2} + 2 \cos \frac{\pi}{2} + 1 \rightarrow 2 \cos^2 \frac{\pi}{2} + 2 \cos \frac{\pi}{2} = 0$$

$$\cos^2 \frac{\pi}{2} + \cos \frac{\pi}{2} = 0 \rightarrow \cos \frac{\pi}{2} (1 + \cos \frac{\pi}{2}) = 0 \rightarrow \cos \frac{\pi}{2} = 0 \rightarrow \frac{\pi}{2} = 2k\pi \pm \frac{\pi}{2} \rightarrow \pi = 4k\pi \pm \pi$$

$$\rightarrow \frac{\pi}{2} = 2k\pi + \pi \rightarrow \pi = 4k\pi + 2\pi \rightarrow \pi = 2\pi$$

ع.ف:  $\{-\pi, 0, \pi\} \rightarrow |\pi - (-\pi) - 0| = |2\pi| = 2\pi$

$$\cos^2 \pi - \sin^2 \pi \cos 3\pi = 1 \rightarrow (\cos^2 \pi - 1) - \sin^2 \pi \cos 3\pi = 0$$

$$\sin^2 \pi - \sin^2 \pi \cos 3\pi = 0 \rightarrow \sin^2 \pi (1 - \cos 3\pi) = 0 \rightarrow \sin^2 \pi = 0$$

$$1 - \cos 3\pi = 0 \rightarrow \cos 3\pi = 1$$

$$\sin \pi = 0 \rightarrow \pi = 2k\pi, \pi = 2k\pi + \pi$$

$$\cos 3\pi = 1 \rightarrow 3\pi = 2k\pi \rightarrow \pi = \frac{2k\pi}{3}$$

ع.ف:  $\{\pi, \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi\} \rightarrow 5$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \quad \tan \alpha = 1 \quad \tan(\alpha - \pi) = \frac{1 - \tan \pi}{1 + \tan \pi}$$

$$\tan\left(\frac{\pi}{4} - \pi\right) = \tan 3\pi$$

$$3\pi = k\pi + \frac{\pi}{4} - \pi \rightarrow 4\pi = k\pi + \frac{\pi}{4} \rightarrow \pi = \frac{k\pi}{4} + \frac{\pi}{16}$$

$$\cos(\sin^2 \pi - \cos^2 \pi)(\sin^2 \pi + \cos^2 \pi) = 1 \rightarrow -\cos 2\pi = 1 \rightarrow \cos 2\pi = -1$$

$$2\pi = 2k\pi + \pi \rightarrow \pi = k\pi + \frac{\pi}{2} \rightarrow \text{ع.ف: } \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

$$\sin^3 \pi - \cos^3 \pi = (\sin \pi - \cos \pi)(\sin^2 \pi + \sin \pi \cos \pi + \cos^2 \pi) = -1$$