۱۷٫۲۵

نام و نام خانوادگی ..رها بدرالی.................... پاسخنامه تشریحی تکلیف شماره .۱۶.... کلاس .دوازدهم دختر B.........

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الف) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0 \quad \Delta = 25$

$\sin x = \frac{-3 \pm 5}{4}$ → -2 غ.ق.ق ، → $\frac{1}{2}$ ق.ق

$\sin x = \sin\frac{\pi}{6} \to x = 2k\pi + \frac{\pi}{6}$ ، $x = 2k\pi + \pi - \frac{\pi}{6}$

ب) $\cos^2 x - \sin^2 x + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0 \quad \Delta = 25$

$\cos x = \frac{-1 \pm 5}{4}$ → 1 ، → $-\frac{3}{2}$ غ.ق.ق

$x = 2k\pi$

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الف) $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\cancel{\sin^2 x + \cos^2 x}) = \sin 4x \to -\cos 2x = 2\sin 2x \cos 2x$

$\to -2\sin 2x = 1 \to \sin 2x = -\frac{1}{2} \to \sin 2x = \sin(-\frac{\pi}{6}) \to$ $2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$ ، $2x = 2k\pi + \pi + \frac{\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

ب) $\underbrace{2\cos^2 x - 1}_{\cos 2x} + \underbrace{2\sin x \cos x}_{\sin 2x} = 0 \to \cos 2x + \sin 2x = 0 \xrightarrow{\wedge 2} \underbrace{\cos^2 2x + \sin^2 2x}_{1} + \underbrace{2\sin 2x \cos 2x}_{\sin 4x} = 0$

$\sin 4x = -1 \to 4x = 2k\pi + \frac{3\pi}{2} \to x = \frac{k\pi}{2} + \frac{3\pi}{8}$

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الف) $\frac{\cos x}{\sin x}(2\cos x + 1) = \frac{1}{\sin x} \to \frac{2\cos^2 x + \cos x}{\sin x} = \frac{1}{\sin x} \to 2\cos^2 x + \cos x = 1$

$\to 2\cos^2 x + \cos x - 1 = 0 \to -1 \to x = 2k\pi + \pi$ ×

$\to \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده میشود
پس جواب غیر قابل قبول محسوب میشود

ب) $\sin^2 x - \sin x \cos x = 1 \to -\sin x \cos x = \cos^2 x \Rightarrow \cos^2 x + \sin x \cos x = 0$

$\cos x(\cos x + \sin x) = 0 \to \cos x = 0 \to x = 2k\pi \pm \frac{\pi}{2}$

$\cos x + \sin x = 0 \to \cos x = -\sin x \to \tan x = -1 \to x = k\pi - \frac{\pi}{4}$

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الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) \xrightarrow{\frac{\pi}{4} + x + \frac{\pi}{4} - x = \frac{\pi}{2}} \cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x)$

$= \frac{1}{2}\sin(\frac{\pi}{2} + 2x) = \frac{1}{2}\cos 2x = \frac{1}{4} \to \cos 2x = \frac{1}{2} \to 2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x = \cos(\frac{\pi}{2} + 2x) \to \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x)$

$2x - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \Rightarrow$ جواب ندارد

$2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \Rightarrow 4x = 2k\pi - \frac{7\pi}{18} \to x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \to \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$

$= 2\cos 2x + 1 = 1 \to 2\cos 2x = 0 \to 2x = 2k\pi \pm \frac{\pi}{2} \to x = k\pi \pm \frac{\pi}{4}$

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$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \underbrace{\sin^2\frac{x}{2}}_{1-\cos^2\frac{x}{2}} = \cos^2\frac{x}{2} + 2\cos\frac{x}{2} + 1 \to 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0$$

$\cos^2\frac{x}{2} + \cos\frac{x}{2} = 0 \to \cos\frac{x}{2}(\cos\frac{x}{2}+1)$ ۶

$\to \cos\frac{x}{2} = 0 \to \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \to x = 4k\pi \pm \pi \hookrightarrow 2k\pm\pi$

$\to \cos\frac{x}{2} = -1 \to \frac{x}{2} = 2k\pi + \pi \to x = 4k\pi + 2\pi \hookrightarrow x = 2\pi$

ج.ع : $\{-\pi, 0, \pi\} \to |\pi-(-\pi)-0| = |2\pi| = 2\pi$

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$\cos^2x - \sin^2x\cos3x = 1 \to (\cos^2x-1) - \sin^2x\cos3x = 0$

$\sin^2x - \sin^2x\cos3x = 0 \to \sin^2x(1-\cos3x) = 0 \to \sin^2x = 0$

$\hookrightarrow 1-\cos3x = 0 \to \cos3x = 1$

$\sin x = 0 \to x = 2k\pi ,\ x = 2k\pi+\pi$ ۷

$\cos3x = 1 \to 3x = 2k\pi \to x = \frac{2k\pi}{3}$

ج.ع : $\{\pi, \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi\} \to$ 5 جواب دارد.

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$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta} \xrightarrow[\alpha=\frac{\pi}{4}]{\tan\alpha=1} \tan(\alpha-x) = \frac{1-\tan x}{1+\tan x}$

$\tan(\frac{\pi}{4}-x) = \tan3x$ ۸

$3x = k\pi + \frac{\pi}{4} - x \to 4x = k\pi + \frac{\pi}{4} \to x = \frac{k\pi}{4} + \frac{\pi}{16}$

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الف) $(\sin^2x-\cos^2x)(\underbrace{\sin^2x+\cos^2x}_{1}) = 1 \to -\cos2x = 1 \to \cos2x = -1$

$2x = 2k\pi+\pi \to x = k\pi+\frac{\pi}{2} \to$ ج.ع : $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

ب) $\sin^3x - \cos^3x = (\sin x-\cos x)(\underbrace{\sin^2x+\sin x\cos x+\cos^2x}_{1}) = -1$ ۱۰

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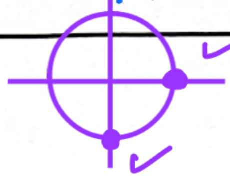
در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 فقط کافی است نقاط اصلی را چک کنی!

$\sin^3x - \cos^3x = 1$



$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \to 2\sin^2 2x + \sin 2x - 1 = 0$$

$\to \sin 2x = -1$

$\to \sin 2x = +\frac{1}{2}$

$$\sin 2x = -1 \to 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \le x \le \pi} x = \frac{3\pi}{4} \checkmark$$

$$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \le x \le \pi} x = \frac{\pi}{12} \checkmark \text{ یا } \frac{5\pi}{12} \times$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ‌ق‌ق است

★ معادله ۲ جواب دارد