۱۲/۵

تکلیف این هفته ساده بود

۲ اگر درصورت وقت گذاشتن اشکال دارید تا یم رفع اشکال بگیرید!

(۱) الف)

$2\sin^2 x - \cos 2x + 1 = 0 \Rightarrow 2\sin^2 x - (1 - 2\sin^2 x) + 1 = 0$

$4\sin^2 x + 2\sin x - 2 = 0$

$\Rightarrow \sin x = \begin{cases} \frac{1}{2} \\ -\frac{2}{2} \times \end{cases}$

$x = \begin{cases} 2k\pi + \frac{\pi}{6} \\ 2k\pi + \frac{5\pi}{6} \end{cases}$

ب)

$2\cos^2 x + \cos x - 1 = 0$

$\cos 2x + 1 + \cos x - 1 = 0$

$\Rightarrow \cos x = \begin{cases} 1 \\ -\frac{1}{2} \times \end{cases}$

$x = 2k\pi$

(۲) الف)

$-\cos 2x = \sin 4x$

$\cos(\pi - 2x) = \cos\left(\frac{\pi}{2} - 4x\right)$

$\pi - 2x = 2k\pi + \frac{\pi}{2} - 4x$

$\Rightarrow x = k\pi - \frac{\pi}{4}$

$\pi - 2x = 2k\pi - \frac{\pi}{2} + 4x$

$\Rightarrow x = \frac{k\pi}{3} + \frac{\pi}{6}$

ب)

$\div \cos^2 x \longrightarrow 2 + 2\tan x = \frac{1}{\cos^2 x}$

$\frac{1}{\cos^2 x} = 1 + \tan^2 x$

$\Rightarrow \tan^2 x - 2\tan x - 1 = 0$

(۳) الف)

$\sin x \neq 0$

$\times \sin x \longrightarrow 2\cos^2 x + \cos x - 1 = 0$

$\Rightarrow \cos x = \begin{cases} -1 \\ \frac{1}{2} \end{cases} \quad \sin x = 0 \times$

$\Rightarrow x = 2k\pi \pm \frac{\pi}{3}$

ب)

$\div \sin^2 x \longrightarrow 2 - \cot x + \cot^2 x = \frac{2}{\sin^2 x}$

$\frac{2}{\sin^2 x} = 2 + 2\cot^2 x$

$\Rightarrow \cot^2 x + \cot x = 0$

$\Rightarrow \cot x = \begin{cases} 0 \\ -1 \end{cases} \Rightarrow x = k\pi + \frac{\pi}{2}$

$\Rightarrow x = k\pi - \frac{\pi}{4}$

(۴) الف)

$\cos\left(x - \frac{\pi}{4}\right) \cos\left(\frac{\pi}{2} + x - \frac{\pi}{4}\right) = \frac{1}{4}$

$-\sin\left(x - \frac{\pi}{4}\right)$

$\times -2 \longrightarrow \sin\left(2x - \frac{\pi}{2}\right) = -\frac{1}{2}$

$2x - \frac{\pi}{2} = 2k\pi - \frac{\pi}{6} \Rightarrow x = k\pi + \frac{\pi}{6}$

$2x - \frac{\pi}{2} = 2k\pi + \frac{7\pi}{6} \Rightarrow x = k\pi + \frac{5\pi}{6}$

ب)

$\cos\left(2\pi - \frac{\pi}{9}\right) = \cos\left(\frac{\pi}{2} + 2x\right)$

$2\pi - \frac{\pi}{9} = 2k\pi + \frac{\pi}{2} + 2x \Rightarrow x = k\pi + \frac{25\pi}{36}$

$2\pi - \frac{\pi}{9} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow x = k\pi - \frac{49\pi}{36}$

(۵)

$\frac{\sin 2x\,(2\cos 2x + 1)}{\sin 2x} = 4\cos^2 x - 1$

$4\cos^2 x + \cos 2x - 2 = 4\cos^2 x - 1$

$\Rightarrow \cos^2 x = \frac{1}{2} \quad x = k\pi \pm \frac{\pi}{4}$

پریا کشتیاری روشن

(۷)

$1-\sin^2 x-\sin^2 x\cos^2 x=1$

$-\sin^2 x(1+\cos^2 x)=0$

$\Rightarrow \sin^2 x=0$

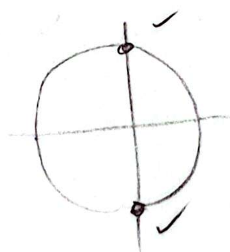
$\Rightarrow \cos^2 x=-1$

$0,\ \frac{\pi}{۲},\ \pi,\ \frac{۵\pi}{۲},\ ۲\pi$

۵ عدد

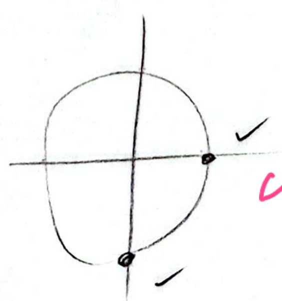
(۱۰)

الف)



$\frac{\pi}{۲},\ \frac{۳\pi}{۲}$

ب)



$0,\ \frac{۳\pi}{۲},\ ۲\pi$

سوال ۲ قسمت ب

$$2\cos^2 x + 2\sin x \cos x = 1 \qquad \begin{cases} 2\sin x \cos x = \sin 2x \\ \dfrac{1+\cos 2x}{2} = \cos^2 x \end{cases}$$

$$1 + \cos 2x + \sin 2x = 1 \rightarrow \cos 2x + \sin 2x = 0$$

$$\tan 2x = -1 \rightarrow 2x = k\pi - \frac{\pi}{4} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$$

سوال ۵

$$\frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x = 1$$

$$\frac{\sin 2x (2\cos 2x + 1)}{\sin 2x} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال ۴

$$\frac{\sin \frac{x}{2}}{1+\cos \frac{x}{2}} = \tan \frac{x}{4}$$

$$\frac{1}{\sin \frac{x}{2}} + \cot \frac{x}{2} = \frac{1+\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1}{\tan \frac{x}{4}} = \cot \frac{x}{4}$$

$$\tan \frac{x}{4} = \cot \frac{x}{4} \rightarrow \tan \frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

جواب‌ها

$k = 0 \rightarrow x = \pi$

$k = -1 \rightarrow x = -\pi$ $\rightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۸

$\tan\frac{\pi}{4}=1 \rightarrow \frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)$

$\tan\left(\frac{5\pi}{4}-x\right)=\tan 5x \rightarrow \frac{\pi}{4}-x=k\pi+3x$

$-4x=k\pi-\frac{\pi}{4} \xrightarrow{-k\pi=k\pi} 4x=k\pi+\frac{\pi}{4}$

$$x=\frac{k\pi}{4}+\frac{\pi}{16}$$

سوال ۹

$\sqrt{1+\sin 4x}=\sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 4x=2\cos^2 2x$

$1+\sin 4x=2-2\sin^2 2x \rightarrow 2\sin^2 2x+\sin 2x-1=0$ ← $\sin 2x=-1$

$\sin 2x=-1 \rightarrow 2x=2k\pi-\frac{\pi}{2} \xrightarrow[k=1]{\text{جایگذاری}} x=\frac{3\pi}{4}$ ✓

$\sin 2x=\frac{1}{2} \rightarrow 2x=2k\pi+\frac{\pi}{6}$ یا $2k\pi+\frac{5\pi}{6} \xrightarrow[k=0]{\text{جایگذاری}} x=\frac{\pi}{12}$ یا $\frac{5\pi}{12}$ ✓ ✗

چون عبارت به توان ۲ رسیده است جواب زائد تولید میکند

به ازای $x=\frac{5\pi}{12}$ عضو $\cos 2x$ صفر میشود پس جواب را نمیپذیرد

* معادله ۲ جواب دارد