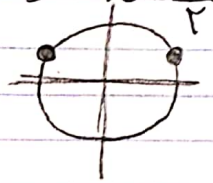
تکلیف شماره ۱۴ / مساجاتی

① الف) $\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{-3 \pm \sqrt{9+16}}{4} = \begin{cases} \frac{-3+5}{4} = \frac{1}{2} \checkmark \\ \frac{-3-5}{4} = -2 \times \end{cases}$

$\sin x = \frac{1}{2} \rightarrow x = 2kn + \frac{n}{6}, 2kn + \frac{5n}{6}$



ب) $\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2\cos^2 x + \cos x - 3 = 0 \xrightarrow{a+b+c=0}$

$\cos x = 1 \rightarrow x = 2kn$

$\cos x = \frac{-3}{2} \times$

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② الف) $\sin^2 x - \cos^2 x = \sin 4x$

$\sin^2 x - \cos^2 x = \sin 4x \rightarrow -\cos 2x = 2\sin 2x \cos 2x \rightarrow$

$\cos 2x (2\sin 2x + 1) = 0$

$\cos 2x = 0 \xrightarrow{\cos \frac{n}{2}} 2x = 2kn \pm \frac{n}{2} \rightarrow x = kn \pm \frac{n}{4}$

$\sin 2x = -\frac{1}{2} \xrightarrow{\sin -\frac{n}{6}} \begin{cases} 2x = 2kn - \frac{n}{6} \rightarrow x = kn - \frac{n}{12} \\ 2x = 2kn + n + \frac{n}{6} \rightarrow x = kn + \frac{7n}{12} \end{cases}$

$\frac{7n}{6}$

ب) $2\cos^2 x + 2\sin x \cos x = 1$

$\cos 2x + \sin 2x = 0 \rightarrow \cos 2x = -\sin 2x \rightarrow \cos 2x = -\cos\left(\frac{\pi}{2} - 2x\right)$

$\cos 2x = \cos\left(\pi - \frac{\pi}{2} + 2x\right) \rightarrow 2x = 2k\pi + \frac{\pi}{2} + 2x$ ✗

$\hookrightarrow 2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

۳)

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$

$a + c = b$

$\frac{\cos x(2\cos x + 1) - 1}{\sin x} = 0 \rightarrow \frac{2\cos^2 x + \cos x - 1}{\sin x} = 0 \rightarrow$

$\cos x = -1 \rightarrow x = 2k\pi \pm \pi$ ($\cos\pi$)

$\sin x \neq 0 \rightarrow x \neq 2k\pi$, $x \neq 2k\pi + \pi$

$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$ ($\cos\frac{\pi}{3}$)

$\xrightarrow{\cap} x = 2k\pi,\ 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 2$

($\sin^2 x$) ($1 - \sin^2 x$)

$\sin^2 x - \sin x \cos x - 1 = 0 \rightarrow -\cos^2 x - \sin x \cos x = 0 \rightarrow$

$-\cos x(\cos x + \sin x) = 0 \rightarrow \cos x = 0 \rightarrow x = 2k\pi \pm \frac{\pi}{2}$

$\cos x + \sin x = 0 \rightarrow \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 0 \rightarrow \sin\left(x + \frac{\pi}{4}\right) = 0 \rightarrow$

$\begin{cases} x + \frac{\pi}{4} = 2k\pi \rightarrow x = 2k\pi - \frac{\pi}{4} \\ x + \frac{\pi}{4} = 2k\pi + \pi \rightarrow x = 2k\pi + \pi - \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \end{cases}$

۴

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4}$

$\cos(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=\frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4}$

$\cos 2x=\frac{1}{2}\rightarrow\cos 2x=\cos\frac{\pi}{3}\rightarrow$

$2x=2k\pi+\frac{\pi}{3}\rightarrow x=k\pi+\frac{\pi}{6}$

$2x=2k\pi-\frac{\pi}{3}\rightarrow x=k\pi-\frac{\pi}{6}$

ب) $\cos(2x-\frac{\pi}{6})=-\sin 2x$

$\sin(\frac{11\pi}{18}-2x)=-\sin 2x\rightarrow\sin(2x-\frac{11\pi}{18})=\sin 2x$

$2x-\frac{11\pi}{18}=2k\pi+2x\rightarrow X$

$2x-\frac{11\pi}{18}=2k\pi+\pi-2x\rightarrow 4x=2k\pi+\frac{29\pi}{18}\rightarrow x=\frac{k\pi}{2}+\frac{29\pi}{72}$

۵

$\frac{2\sin 2x\cos 2x}{\sin 2x+\sin 2x}-\sin^2 x=\cos^2 x$

$\frac{\sin 2x(2\cos 2x+1)}{\sin 2x}=1\rightarrow 2\cos 2x+1=1\rightarrow\cos 2x=0$

$2x=2k\pi\pm\frac{\pi}{2}\rightarrow x=k\pi\pm\frac{\pi}{4}$

$\sin 2x\neq 0\rightarrow 2x\neq 2k\pi\rightarrow x\neq k\pi$

$x\neq 2k\pi+\pi\rightarrow x\neq k\pi+\frac{\pi}{2}$

(٩)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} \qquad [-\pi, \frac{3\pi}{2}]$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \sin^2\frac{x}{2} = \cos^2\frac{x}{2} + 2\cos\frac{x}{2} + 1 \quad (1-\cos^2\frac{x}{2})$$

$$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right) = 0$$

$$\cos\frac{x}{2} = 0 \rightarrow \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \rightarrow x = 4k\pi \pm \pi \rightarrow \checkmark$$

$$\cos\frac{x}{2} + 1 = 0 \quad , \quad \cos\frac{x}{2} + 1 \neq 0$$

k	0	1
x	π ✓	3π ✓
	$-\pi$ ✗	-3π ✗

$$\sin\frac{x}{2} \neq 0 \rightarrow \frac{x}{2} \neq 2k\pi \rightarrow x \neq 4k\pi$$

$$\frac{x}{2} \neq 2k\pi + \pi \rightarrow x \neq 4k\pi + 2\pi$$

تدريب تفاعلي $= |\pi + \pi| = 2\pi$

(٧)

$$\cos^2 x - \sin^2 x \cos 2x = 1 \qquad [0, 2\pi]$$

$$1 - \sin^2 x$$

$$-\sin^2 x(1+\cos 2x) = 0 \rightarrow \sin^2 x = 0 \rightarrow x = 2k\pi \;,\; x = 2k\pi + \pi$$

$$1 + \cos 2x = 0 \rightarrow \cos 2x = -1 \rightarrow 2x = 2k\pi \pm \pi \rightarrow x = \frac{2k\pi}{2} \pm \frac{\pi}{2}$$

k	0	1	2
x	2π	4π	6π
	π	3π	5π
	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$

$$x \in \left\{0, \pi, \frac{\pi}{2}, 2\pi, \frac{3\pi}{2}\right\}$$

$$\frac{1-\tan\alpha}{1+\tan\alpha}=\tan^{2}\alpha$$ (۸)

$$\tan\left(\frac{\pi}{4}-\alpha\right)=\frac{\tan\frac{\pi}{4}-\tan\alpha}{1+\tan\frac{\pi}{4}\tan\alpha}=\frac{1-\tan\alpha}{1+\tan\alpha}$$

$$\tan\left(\frac{\pi}{4}-\alpha\right)=\tan^{2}\alpha \rightarrow \frac{\pi}{4}-\alpha=k\pi+2\alpha \rightarrow 3\alpha=-k\pi+\frac{\pi}{4} \rightarrow \alpha=\frac{-k\pi}{3}+\frac{\pi}{12}$$

(۹)

$$\sqrt{1+\sin 2\alpha}=\sqrt{2}\cos 2\alpha \quad [0,\pi]$$

$$\sqrt{(\sin\alpha+\cos\alpha)^{2}}=\sqrt{2}\cos 2\alpha \rightarrow \sin\alpha+\cos\alpha=\sqrt{2}\cos 2\alpha$$

$$\sqrt{2}\sin\left(\alpha+\frac{\pi}{4}\right)=\sqrt{2}\cos 2\alpha \rightarrow$$

$$\cos\left(\frac{\pi}{4}-\alpha\right)=\cos 2\alpha$$

$$\rightarrow \frac{\pi}{4}-\alpha=2k\pi+2\alpha \rightarrow 3\alpha=\frac{\pi}{4}-2k\pi \rightarrow \alpha=\frac{\pi}{12}-\frac{2k\pi}{3}$$

$$\rightarrow \frac{\pi}{4}-\alpha=2k\pi-2\alpha \rightarrow \alpha=2k\pi-\frac{\pi}{4}$$

k	0	1
α	$\frac{\pi}{12}$	$-\frac{7\pi}{12}$
	$-\frac{\pi}{4}$	$\frac{7\pi}{4}$

→ بیضی

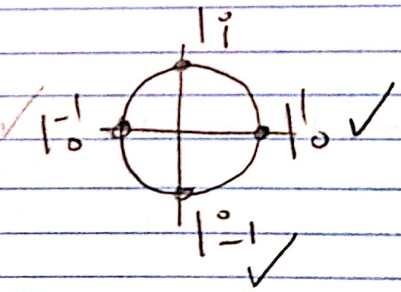
۱۰) گزینه ۱ $\cos\pi$

الف) $\sin^4 x - \cos^4 x = 1 \rightarrow \sin^2 x - \cos^2 x = 1 \rightarrow -\cos 2x = 1 \rightarrow \cos 2x = -1$

$2x = 2k\pi \pm \pi \rightarrow x = k\pi \pm \frac{\pi}{2}$

k	۰	۱	۲
x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$
	$\frac{-\pi}{2}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$

$x \in \{\frac{\pi}{2}, \frac{3\pi}{2}\}$



ب) $\sin^2 x - \cos^2 x = -1$

$x \in \{0, \frac{3\pi}{2}\}$