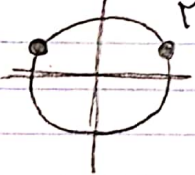


الف) $\cos 2x = 3 \sin x - 1$ (۱)

$$1 - 2 \sin^2 x = 3 \sin x - 1 \rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0$$

$$\sin x = \frac{-3 \pm \sqrt{9 + 16}}{4} = \begin{cases} \frac{-3 + 5}{4} = \frac{1}{2} \checkmark \\ \frac{-3 - 5}{4} = -2 \times \end{cases}$$

$$\sin x = \frac{1}{2} \rightarrow x = 2kn + \frac{\pi}{6}, 2kn + \frac{5\pi}{6}$$



ب) $\cos 2x + \cos x - 2 = 0$

$$2 \cos^2 x - 1 + \cos x - 2 = 0 \rightarrow 2 \cos^2 x + \cos x - 3 = 0 \quad a+b+c=0 \rightarrow$$

$$\cos x = 1 \rightarrow x = 2kn$$

$$\cos x = \frac{-1 \pm \sqrt{1 + 24}}{4} \times$$

الف) $\sin^2 x - \cos^2 x = \sin 4x$ (۲)

$$\sin^2 x - \cos^2 x = \sin 4x \rightarrow -\cos 2x = 2 \sin^2 x \cos 2x \rightarrow$$

$$\cos 2x (2 \sin^2 x + 1) = 0 \rightarrow \cos 2x = 0$$

$$\cos 2x = 0 \rightarrow 2x = 2kn + \frac{\pi}{2} \rightarrow x = kn + \frac{\pi}{4}$$

$$\sin 2x = -\frac{1}{2} \rightarrow \begin{cases} 2x = 2kn - \frac{\pi}{6} \rightarrow x = kn - \frac{\pi}{12} \\ 2x = 2kn + \frac{7\pi}{6} \rightarrow x = kn + \frac{7\pi}{12} \end{cases}$$

ب) $\sqrt{2} \cos x + \sqrt{2} \sin x \cos x = 1$

$\sqrt{2} \cos x + \sin x = 0 \rightarrow \cos x = -\sin x \rightarrow \cos x = -\cos(\frac{\pi}{2} - x)$

$\cos x = \cos(\frac{\pi}{2} - x) \rightarrow x = \frac{\pi}{2} - x + 2k\pi$

$x = \frac{\pi}{2} - x + 2k\pi \rightarrow x = \frac{\pi}{2} + k\pi$

الف) $\cot x (\sqrt{2} \cos x + 1) = \frac{1}{\sin x}$

$\frac{\cos x (\sqrt{2} \cos x + 1) - 1}{\sin x} = 0 \rightarrow \sqrt{2} \cos^2 x + \cos x - 1 = 0$

$\cos x = -\frac{1}{\sqrt{2}} \rightarrow x = \frac{3\pi}{4} + 2k\pi$ $\sin x \neq 0 \rightarrow x \neq k\pi$

$\cos x = \frac{1}{\sqrt{2}} \rightarrow x = \frac{\pi}{4} + 2k\pi$ $\frac{\pi}{4} \rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$

ب) $\sqrt{2} \sin^2 x - \sin x \cos x + \cos^2 x = \sqrt{2}$

$\sin^2 x - \sin x \cos x - 1 = 0 \rightarrow -\cos^2 x - \sin x \cos x = 0$

$-\cos x (\cos x + \sin x) = 0 \rightarrow \cos x = 0 \rightarrow x = \frac{\pi}{2} + k\pi$

$\cos x + \sin x = 0 \rightarrow \sqrt{2} \sin(x + \frac{\pi}{4}) = 0 \rightarrow \sin(x + \frac{\pi}{4}) = 0$

$x + \frac{\pi}{4} = k\pi \rightarrow x = k\pi - \frac{\pi}{4}$

$x + \frac{\pi}{4} = k\pi + \pi \rightarrow x = k\pi + \pi - \frac{\pi}{4} = k\pi + \frac{3\pi}{4}$

(۴)

الف) $\cos\left(\frac{n}{r} + x\right) \cos\left(x - \frac{n}{r}\right) = \frac{1}{r}$

$$\cos\left(x + \frac{n}{r}\right) \sin\left(x + \frac{n}{r}\right) = \frac{1}{r} \sin\left(2x + \frac{n}{r}\right) = \frac{1}{r}$$

$$\cos 2x = \frac{1}{r} \rightarrow \cos 2x = \cos \frac{n}{r} \rightarrow$$

$$2x = 2kn + \frac{n}{r} \rightarrow x = kn + \frac{n}{2r} \quad \checkmark$$

$$2x = 2kn - \frac{n}{r} \rightarrow x = kn - \frac{n}{2r} \quad \checkmark$$

ب) $\cos\left(2x - \frac{n}{r}\right) = -\sin 2x$

$$\sin\left(\frac{11n}{18} - 2x\right) = -\sin 2x \rightarrow \sin\left(2x - \frac{11n}{18}\right) = \sin 2x$$

$$2x - \frac{11n}{18} = 2kn + 2x \rightarrow \times$$

$$2x - \frac{11n}{18} = 2kn + n - 2x \rightarrow 4x = 2kn + \frac{19n}{18} \rightarrow x = \frac{kn}{2} + \frac{19n}{36} \quad \checkmark$$

$$\frac{y \sin 2x \cos 2x}{\sin 2x + \sin 2x} - \sin^2 x = \cos^2 x$$

$$\sin 2x$$

$$\frac{\sin 2x (y \cos 2x + 1)}{\sin 2x} = 1 \rightarrow y \cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = 2kn + \frac{n}{2} \rightarrow x = kn + \frac{n}{4} \quad \checkmark$$

$$\sin 2x \neq 0 \rightarrow 2x \neq 2kn \rightarrow x \neq kn \quad \checkmark$$

$$x \neq 2kn + n \rightarrow x \neq kn + \frac{n}{2} \quad \checkmark$$

(9)

$$\frac{\sin \frac{x}{p}}{1 + \cos \frac{x}{p}} = \frac{1}{\sin \frac{x}{p}} + \cot \frac{x}{p} \quad [-n, \frac{2n}{p}]$$

$$\frac{\sin \frac{x}{p}}{1 + \cos \frac{x}{p}} = \frac{1 - \cos^2 \frac{x}{p}}{\sin \frac{x}{p}} \Rightarrow \frac{\sin \frac{x}{p}}{1 + \cos \frac{x}{p}} = \cos^2 \frac{x}{p} + p \cos \frac{x}{p} + 1 \rightarrow$$

$$p \cos^2 \frac{x}{p} + p \cos \frac{x}{p} = 0 \rightarrow \cos \frac{x}{p} (\cos \frac{x}{p} + 1) = 0$$

$$\cos \frac{x}{p} = 0 \rightarrow \frac{x}{p} = pkn \pm \frac{\pi}{2} \rightarrow x = pkn \pm \frac{\pi}{2} \rightarrow \checkmark$$

$$\cos \frac{x}{p} + 1 = 0, \cos \frac{x}{p} + 1 \neq 0$$

k	0	1
qx	n	wnx
	-n	pnx

$$\sin \frac{x}{p} \neq 0 \Rightarrow \frac{x}{p} \neq pkn \rightarrow x \neq pkn$$

$$\text{تدقيق } |n+n| = pn$$

$$\frac{x}{p} \neq pkn + \pi \rightarrow x \neq pkn + \pi$$

$$\cos^2 x - \sin^2 x \cos^2 x = x \quad [0, \pi]$$

(10)

$$-\sin^2 x (1 + \cos^2 x) = 0 \rightarrow \sin^2 x = 0 \rightarrow x = pkn, x = pkn + \pi$$

$$1 + \cos^2 x = 0 \rightarrow \cos^2 x = -1 \rightarrow x = pkn + \frac{\pi}{2} \rightarrow x = \frac{pkn}{p} + \frac{\pi}{p}$$

k	0	1	2
x	0	pn	n
	n	pn	wn
	n	n	wn
	pn	pn	n

$$x \in \{0, \pi, \frac{\pi}{p}, \pi, \frac{wn}{p}\}$$

$$\frac{1 - \tan x}{1 + \tan x} = \tan^2 x$$

①

$$\tan\left(\frac{n}{2} - x\right) = \frac{\tan \frac{n}{2} - \tan x}{1 + \tan \frac{n}{2} \tan x} = \frac{1 - \tan x}{1 + \tan x}$$

②

$$\tan\left(\frac{n}{2} - x\right) = \tan^2 x \rightarrow \frac{n}{2} - x = kn + 2x \rightarrow 3x = -kn + \frac{n}{2} \rightarrow x = \frac{-kn}{3} + \frac{n}{6}$$

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos x \quad [0, \pi]$$

③

$$\sqrt{(\sin x + \cos x)^2} = \sqrt{2} \cos 2x \rightarrow \sin x + \cos x = \sqrt{2} \cos 2x$$

④

$$\sqrt{\sin\left(x + \frac{n}{2}\right)} = \sqrt{\cos 2x} \rightarrow$$

$$\cos\left(\frac{n}{2} - x\right) = \cos 2x \rightarrow \frac{n}{2} - x = 2kn + 2x$$

$$\rightarrow \frac{n}{2} - x = 2kn + 2x \rightarrow 3x = \frac{n}{2} - 2kn \rightarrow x = \frac{n}{6} - \frac{2kn}{3}$$

$$\rightarrow \frac{n}{2} - x = 2kn - 2x \rightarrow x = 2kn - \frac{n}{2}$$

k	0	1
x	$\frac{n}{6}$	$-\frac{2n}{3}$
	$\frac{n}{6}$	$\frac{2n}{3}$

→ جواب

$$\sqrt{1 + \sin 2\pi} = \sqrt{2} \cos \pi \xrightarrow{\text{توان ۲}} 1 + \sin 2\pi = 2 \cos^2 \pi$$

مسئله ۹

$$1 + \sin 2\pi = 2 - 2 \sin^2 \pi \rightarrow 2 \sin^2 \pi + \sin 2\pi - 1 = 0 \rightarrow \sin 2\pi = -1$$

$$\sin 2\pi = -1 \rightarrow 2\pi = k\pi - \frac{\pi}{2} \xrightarrow{\cdot \frac{2}{k\pi}} \pi = \frac{k\pi}{2} - \frac{\pi}{4}$$

$$\sin 2\pi = \frac{1}{2} \rightarrow 2\pi = k\pi + \frac{\pi}{6} \text{ یا } 2\pi = k\pi + \frac{5\pi}{6} \xrightarrow{\cdot \frac{2}{k\pi}} \pi = \frac{k\pi}{3} + \frac{\pi}{12} \text{ یا } \frac{k\pi}{3} + \frac{5\pi}{12}$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند
به ازای $\pi = \frac{5\pi}{12}$ $\cos 2\pi$ منتهی به $\frac{1}{2}$ پس جواب را باید از آن حذف کرد

و معادله ۲ جواب دارد

دی ۹۴

2 JAN 2016 ۲۱ ربیع الاول ۱۴۳۷

شنبه ۱۲

مسئله ۱۰

الف) $\sin^2 x - \cos^2 x = 1 \rightarrow \sin^2 x - \cos^2 x = 1 \rightarrow -\cos 2x = 1 \rightarrow \cos 2x = -1$

$$2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2}$$

k	0	1	2
x	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$
	$-\frac{\pi}{2}$	$-\frac{3\pi}{2}$	$-\frac{5\pi}{2}$

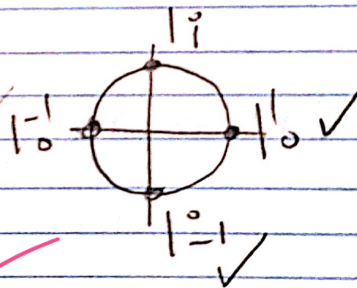
$$x \in \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

۱، ۳، ۵

ب) $\sin^2 x - \cos^2 x = -1$

$$x \in \left\{ 0, \frac{2\pi}{2} \right\}$$

۲، ۴



در معادلات به فرم $\pm \sin^2 x \pm \cos^2 x$ برابر ۱ یا -۱. فقط نامی است نگاه منور!

$$\sin^2 x - \cos^2 x = 1$$

$$x = 2k\pi \text{ یا } 0$$

$$x = \frac{2\pi}{2}$$

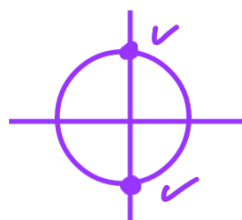


ج ۱ چک کن!

$$\sin^2 x - \cos^2 x = -1$$

$$x = \frac{\pi}{2}$$

$$x = \frac{3\pi}{2}$$



الف ۱