دوازدهم دختر B

به نام خدا

هستی کرمی
شماره ی تکلیف : ۱۴

1

الف) $\cos 2x = 3\sin x - 1 \Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow$

$\underbrace{2\sin^2 x + 3\sin x - 2 = 0}$ $\longrightarrow \sin^2 x + 3\sin x - 4 = 0 \Rightarrow$

$(\sin x + 4)(\sin x - 1) = 0 \xrightarrow{\div 2} \sin x = \frac{1}{2}$

$\sin x = -2$ غ ق ق

$\sin x = \frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6} \Rightarrow \boxed{x = 2k\pi + \frac{\pi}{6}}$ $k \in \mathbb{Z}$

ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0 \Rightarrow \cos^2 x + \cos x - 6 = 0$

$(\cos x + 3)(\cos x - 2) = 0 \xrightarrow{\div 2} \cos x = \frac{-3}{2}$ غ ق ق

$\cos x = 1 \longrightarrow \cos x = \cos 2\pi \Rightarrow$

$x = 2k\pi \pm 2\pi$

$\boxed{x = 2k\pi}$

2

الف) $\sin^4 x - \cos^4 x = \sin 4x \Rightarrow -\cancel{\cos 2x} = 2\sin 2x \cancel{\cos 2x} \Rightarrow$

$2\sin 2x = -1 \Rightarrow \sin 2x = \frac{-1}{2}$

$\Rightarrow \sin 2x = \sin\frac{-\pi}{6}$

$2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}}$ ✓

$2x = 2k\pi + \pi + \frac{\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}}$ ✓

یکی از جواب ها

$\cos 2x = 0 \Rightarrow \cos 2x = \cos\frac{\pi}{2} \Rightarrow$

$2x = 2k\pi \pm \frac{\pi}{2} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{4}}$

$k \in \mathbb{Z}$

جواب کلی : $\frac{k\pi}{2} + \frac{\pi}{4}$

ب) $2\cos^2 x + \underbrace{2\sin x\cos x}_{\sin 2x} = 1 \Rightarrow \sin 2x = 1 - \underbrace{2\cos^2 x}_{-\cos 2x} \Rightarrow \sin 2x = -\cos 2x$

$\xrightarrow{\div \cos 2x \neq 0} \tan 2x = -1 \Rightarrow \tan 2x = \tan\frac{-\pi}{4} \Rightarrow 2x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

③

الف) $\cot x(۲\cos x+۱) = \frac{۱}{\sin x} \Rightarrow \frac{\cos x}{\cancel{\sin x}}(۲\cos x+۱) = \frac{۱}{\sin x} \Rightarrow$

$\boxed{۲\cos^۲ x+\cos x-۱}=۰ \to \cos x=-۱ \Rightarrow \cos x=\cos\pi \Rightarrow x=۲k\pi\pm\pi$

$\to \cos x=\frac{۱}{۲} \Rightarrow \cos x=\cos\frac{\pi}{۳} \Rightarrow x=۲k\pi\pm\frac{\pi}{۳}$

ب) $۲\sin^۲ x-\sin x\cos x+\cos^۲ x=۲ \to \sin^۲+\sin^۲+\cos^۲-\sin x\cdot\cos x=۲$

$\sin^۲ x-\frac{۱}{۲}\sin ۲x=۱ \Rightarrow \cos^{\cancel{۲}} x=-\sin x\,\cancel{\cos x} \Rightarrow \cos x=-\sin x$

$\xrightarrow{\div\cos x} \tan x=-۱=\tan\frac{-\pi}{۴} \Rightarrow$

$x=k\pi-\frac{\pi}{۴}$

$\underline{\cos x=۰}=\cos\frac{\pi}{۲} \Rightarrow$

$x=۲k\pi\pm\frac{\pi}{۲}$

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④

$\cos(x-\frac{\pi}{۴})=\cos(-(x-\frac{\pi}{۴}))$

الف) $\cos(\frac{\pi}{۴}+x)\cos(x-\frac{\pi}{۴})=\frac{۱}{۴} \Rightarrow \cos(x+\frac{\pi}{۴})\cos(\frac{\pi}{۴}-x)=\frac{۱}{۴}$

متمم اند

$\cos(x+\frac{\pi}{۴})\sin(x+\frac{\pi}{۴})=\frac{۱}{۴} \Rightarrow \frac{۱}{۲}\sin(۲x+\frac{\pi}{۲})=\frac{۱}{۴} \Rightarrow \cos ۲x=\frac{۱}{۲}$

$\Rightarrow \cos ۲x=\cos\frac{\pi}{۳} \Rightarrow ۲x=۲k\pi\pm\frac{\pi}{۳} \Rightarrow x=k\pi\pm\frac{\pi}{۶}$

ب) $\left.\begin{array}{l}\cos(۲x-\frac{\pi}{۹})=-\sin ۲x \\ -\sin ۲x=\cos(\frac{\pi}{۲}+۲x)\end{array}\right\} \Rightarrow \cos(۲x-\frac{\pi}{۹})=\cos(\frac{\pi}{۲}+۲x) \Rightarrow$

$۲x-\frac{\pi}{۹}=۲k\pi\pm(\frac{\pi}{۲}+۲x)$

$\to \cancel{۲x}-\frac{\pi}{۹}=۲k\pi+\frac{\pi}{۲}+\cancel{۲x} \Rightarrow$

$۲k\pi=\frac{-۲\pi}{۱۸}-\frac{۹\pi}{۱۸}=\frac{-۱۰\pi}{۱۸}$ غ ق ق

$\to ۲x-\frac{\pi}{۹}=۲k\pi-\frac{\pi}{۲}-۲x$

$۴x=۲k\pi-\frac{۹\pi}{۱۸}+\frac{۲\pi}{۱۸} \Rightarrow$

$۴x=۲k\pi-\frac{۷\pi}{۱۸} \Rightarrow$

$x=\frac{k\pi}{۲}-\frac{۷\pi}{۷۲}$

⑤

$$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \Rightarrow \frac{2\sin 2x \cos 2x + \sin 2x}{\sin 2x} = \cos 2x$$

$$2\cos 2x + 1 = \cos 2x \Rightarrow \cos 2x = -1 \Rightarrow \cos 2x = \cos \pi \Rightarrow 2x = 2k\pi \pm \pi$$

اما در مجموعه جواب داده شده مقدار $\sin 2x$ که در مخرج می باشد صفر می شود و تعریف نشده است پس این معادله جوابی ندارد.

$[-\pi, \frac{3\pi}{2}]$

⑥

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \xrightarrow[\text{وسطین}]{\text{طرفین}} \underbrace{\sin^2\frac{x}{2}}_{1-\cos^2\frac{x}{2}} = \cos^2\frac{x}{2} + 2\cos\frac{x}{2} + 1 \Rightarrow$$

$$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0 \Rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2} + 1\right) = 0$$

$\rightarrow \cos\frac{x}{2} = 0 = \cos\frac{\pi}{2}$

$\rightarrow \cos\frac{x}{2} = -1 = \cos\pi$

$$\left.\begin{array}{l} \frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \Rightarrow x = 4k\pi \pm \pi \\ \frac{x}{2} = 2k\pi \pm \pi \Rightarrow x = 4k\pi \pm 2\pi \end{array}\right\} \xrightarrow[{[-\pi, \frac{3\pi}{2}]}]{\text{جواب های بازه}} -\pi, \pi \Rightarrow |\pi + \pi| = \boxed{2\pi}$$

جواب آخر

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⑦

$[0, 2\pi]$

$$\cos^2 x - \sin^2 x \cdot \cos^3 x = 1 \Rightarrow \cos^2 x - 1 = \sin^2 x \cdot \cos^3 x \Rightarrow$$

$$-\cancel{\sin^2 x} = \cancel{\sin^2 x} \cdot \cos^3 x \Rightarrow \cos^3 x = -1 \Rightarrow \cos 3x = \cos\pi \Rightarrow$$

$$3x = 2k\pi \pm \pi \Rightarrow$$

$$\boxed{x = \frac{2k\pi}{3} \pm \frac{\pi}{3}}$$

$$\sin^2 x = 0 \Rightarrow \sin x = 0 \Rightarrow \sin x = \sin\pi$$

$$\boxed{\begin{array}{l} x = 2k\pi + \pi \\ x = 2k\pi \end{array}} \Rightarrow \boxed{x = k\pi}$$

$K = 0 \rightarrow \boxed{x = 0}$, $\boxed{x = \frac{\pi}{3}}$

$K = 1 \rightarrow \boxed{x = \pi}$, $x = \frac{\pi}{3}$ ✓

$K = 2 \rightarrow \boxed{x = 2\pi}$, $\boxed{x = \frac{5\pi}{3}}$

۵ جواب دارد

⑧

$\cos \neq 0$

$$\frac{1-\tan x}{1+\tan x} = \tan ۳x \Rightarrow \tan(\frac{\pi}{۴}-x) = \tan ۳x \Rightarrow$$

$$۳x = k\pi + \frac{\pi}{۴} - x \Rightarrow ۴x = k\pi + \frac{\pi}{۴} \Rightarrow \underbrace{x = \frac{k\pi}{۴} + \frac{\pi}{۱۶}}\ \Big\}\ k \in \mathbb{Z}$$

محاسبه.

$$\boxed{\frac{1-\tan x}{1+\tan x} = \tan(\frac{\pi}{۴}-x)}$$

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⑨ $[0, \pi]$

$$\sqrt{1+\sin ۲x} = \sqrt{۲}\cos ۲x \xRightarrow{(\ )^۲} 1+\sin ۲x = ۲\cos^۲ ۲x \Rightarrow \quad \cos^۲ ۲x = 1-\sin^۲ ۲x$$

$$1+\sin ۲x = ۲ - ۲\sin^۲ ۲x \Rightarrow ۲\sin^۲ ۲x + \sin ۲x - 1 = 0 \to \sin ۲x = -1$$

$$\to \sin ۲x = \frac{1}{۲}$$

$$\sin ۲x = -1 \Rightarrow \sin ۲x = \sin\frac{۳\pi}{۲} \Rightarrow ۲x = ۲k\pi + \frac{۳\pi}{۲} \Rightarrow x = k\pi + \frac{۳\pi}{۴}$$

$$۲x = ۲k\pi + \pi - \frac{۳\pi}{۲} \Rightarrow x = k\pi - \frac{\pi}{۴}$$

$$\sin ۲x = \sin\frac{\pi}{۶} \Rightarrow ۲x = ۲k\pi + \frac{\pi}{۶} \Rightarrow x = k\pi + \frac{\pi}{۱۲}$$

$$۲x = ۲k\pi + \frac{۵\pi}{۶} \Rightarrow x = k\pi + \frac{۵\pi}{۱۲}$$

جواب‌های معادله در بازه‌ی $[0, \pi]$: ۳ جواب . $\frac{۳\pi}{۴}$ ، $\frac{\pi}{۱۲}$ و $\frac{۵\pi}{۱۲}$

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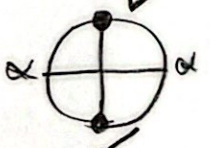
⑩

الف) $\sin^۴ x - \cos^۴ x = 1 \Rightarrow -\cos ۲x = 1 \Rightarrow \cos ۲x = -1 \Rightarrow$

$$\cos ۲x = \cos\pi \Rightarrow ۲x = ۲k\pi \pm \pi \Rightarrow x = k\pi \pm \frac{\pi}{۲}$$



راه ۲) از دایره‌ی مثلثاتی و نقاط اساسی

ب) $\sin^۳ x - \cos^۳ x = -1$

$$x = ۲k\pi$$
$$x = ۲k\pi - \frac{\pi}{۲}$$