پرهیس زندی

سؤال ①

الف) $\cos 2x = 2\sin x - 1$

$1 - 2\sin^2 x = 2\sin x - 1 \Rightarrow -2\sin^2 x - 2\sin x + 2 = 0 \Rightarrow 2\sin^2 x + 2\sin x - 2 = 0$

اگر $\sin x = t \Rightarrow 2t^2 + 2t - 2 = 0 \Rightarrow \frac{-2 \pm \sqrt{9+4}}{4}$ ، $t = \frac{1}{2}$ ✓ ، $t = -2$ ✗ غ.ق.ق

$\Rightarrow \sin x = \frac{1}{2} \Rightarrow \left\{ x \mid x = 2k\pi + \frac{\pi}{6}, x = 2k\pi + \frac{5\pi}{6}, k \in \mathbb{Z} \right\}$ $-1 \le \sin x \le 1$

ب) $\cos 2x + \cos x - 2 = 0$

$\cos 2x = 2\cos^2 x - 1 \Rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x = t \Rightarrow 2t^2 + t - 3 = 0 \Rightarrow \frac{-1 \pm \sqrt{1+24}}{4} \Rightarrow$ ، $t = -\frac{3}{2}$ ✗ غ.ق.ق ، $t = 1$ ✓

$-1 \le \cos x \le 1$

$\cos x = 1 \Rightarrow \boxed{x = 2k\pi}$

سؤال ②

الف) $\sin^4 x - \cos^4 x = \sin 4x$

$\Rightarrow -\cos 2x = \sin 4x$

$(\underbrace{\sin^2 x - \cos^2 x}_{-\cos 2x})(\sin^2 x + \cos^2 x) = \sin 4x$ ($\sin^2 x + \cos^2 x = 1$)

$\Rightarrow 2\sin 2x \cos 2x + \cos 2x = 0$

$\cos 2x(2\sin 2x + 1) = 0$

$\sin 4x = 2\sin 2x \cos 2x$

① $\Rightarrow \cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

② $\Rightarrow 2\sin 2x + 1 = 0 \Rightarrow \sin 2x = -\frac{1}{2} \Rightarrow \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}} \\ 2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}} \end{cases}$

$\Rightarrow \left\{ x = \frac{k\pi}{2} + \frac{\pi}{4}, x = k\pi - \frac{\pi}{12}, x = k\pi + \frac{7\pi}{12} \mid k \in \mathbb{Z} \right\}$

ب) $2\cos^2 x + 2\sin x \cos x = 1 \Rightarrow 2(\cos^2 x + \sin x \cos x) = 1 \Rightarrow$

$\cos^2 x + \sin x \cos x = \frac{1}{2} \Rightarrow 1 - \sin^2 x + \sin x \cos x = \frac{1}{2}$

ضمن ذوزیل $\cos^2 x = \frac{1 + \cos 2x}{2}$ ، $\sin x \cos x = \frac{1}{2}\sin 2x$

$\frac{1 + \cos 2x}{2} + \frac{1}{2}\sin 2x = \frac{1}{2} \Rightarrow 1 + \cos 2x + \sin 2x = 1 \Rightarrow \cos 2x + \sin 2x = 0$

$\sin 2x = -\cos 2x \xrightarrow{\div \cos 2x} \tan 2x = -1 \Rightarrow 2x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

سؤال (۳)

ب) $\cot x(2\cos x+1)=\frac{1}{\sin x}$

$\frac{\cos x}{\sin x}\times(2\cos x+1)=\frac{1}{\sin x} \Rightarrow 2\cos^2 x+\cos x-1=0$

$\cos x=t \Rightarrow 2t^2+t-1=0 \Rightarrow \frac{-1\pm\sqrt{1+8}}{4}=\begin{cases} t=\frac{1}{2} \\ t=-1 \end{cases} \Rightarrow \cos x=-1,\frac{1}{2}$

$\Rightarrow \boxed{\begin{array}{l} x=2k\pi\pm\frac{\pi}{3} \\ x=2k\pi+\pi \end{array}}$

ج) $\underset{a}{2}\sin^2 x \underset{c}{-1}\sin x\cos x+\underset{b}{1}\cos^2 x=\underset{d}{2}$ $\quad a\sin^2 x+b\cos^2 x+c\sin x\cos x=d \quad a=d$

$\div \sin^2 x$

$\Rightarrow 2+\cot^2 x-\cot x=2(1+\cot^2 x) \Rightarrow \cot^2 x+\cot x=0 \Rightarrow$

$\cot x(\cot x+1)=0 \begin{cases} \cot x=0 \rightarrow x=k\pi+\frac{\pi}{2} \\ \cot x=-1 \rightarrow x=k\pi-\frac{\pi}{4} \end{cases}$

سؤال (۴)

ب) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4}$

$\cos A\cos B=\frac{1}{2}(\cos(A+B)+\cos(A-B))=\frac{1}{2}(\cos\frac{\pi}{2}+\cos 2x))$

$A=\frac{\pi}{4}+x \quad B=x-\frac{\pi}{4} \begin{cases} A+B=2x \\ A-B=\frac{\pi}{2} \end{cases}$

$=\frac{1}{2}\cos 2x=\frac{1}{4} \Rightarrow \cos 2x=\frac{1}{2}$

$\Rightarrow 2x=2k\pi\pm\frac{\pi}{3} \Rightarrow \boxed{x=k\pi\pm\frac{\pi}{6}}$

ج) $\cos(2x-\frac{\pi}{6})=-\sin^2 x \overset{(I)}{\Rightarrow} \cos(2x-\frac{\pi}{6})=\cos(2x+\frac{\pi}{2})$

$-\sin^2 x=\cos(\frac{\pi}{2}+2x)$ (I)

$\cos A=\cos B$

سوال ⑤

$$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \Rightarrow \frac{\sin 4x}{\sin 2x}+\frac{\sin 2x}{\sin 2x}=$$

$$\frac{2\sin 2x\cos 2x}{\sin 2x}+1=\boxed{2\cos 2x+1}$$

$$2\cos 2x+1-\sin^2 x=\cos^2 x \Rightarrow 2\cos 2x+1=\cos^2 x+\sin^2 x \Rightarrow \cos 2x=0$$

$$\Rightarrow 2x=k\pi+\frac{\pi}{2} \Rightarrow \boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}} \quad k\in\mathbb{Z}$$

سوال ⑥

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2} \qquad [-\pi,\tfrac{3\pi}{2}]$$

$$\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow$$

$$\sin^2\frac{x}{2}=\left(1+\cos\frac{x}{2}\right)^2 \Rightarrow 1+\cos\frac{x}{2}=\pm\sin\frac{x}{2}$$

$1+\cos\frac{x}{2}=\sin\frac{x}{2}$

$$\Rightarrow \sin\frac{x}{2}-\cos\frac{x}{2}=1 \Rightarrow \begin{cases} x=4k\pi+\frac{\pi}{2} \\ x=4k\pi+\pi \end{cases}$$

$1+\cos\frac{x}{2}=-\sin\frac{x}{2}$

$$\cos\frac{x}{2}+\sin\frac{x}{2}=-1 \Rightarrow \begin{cases} x=4k\pi-\frac{\pi}{2} \\ x=4k\pi+\pi \end{cases}$$

سوال ⑦

$\cos^2 x-\sin^2 x\cos 2x=1$ در فاصله $[0,2\pi]$

$$\frac{1+\cos 2x}{2}-\frac{1-\cos 2x}{2}\times\cos^2 2x=1$$

$$1+\cos^2 2x-(1-\cos 2x)\cos^2 2x=2 \Rightarrow \cos 2x-\cos^2 2x+\cos 2x\cos^2 2x=1$$

$$\cos 2x\left(\cos^2 2x+1\right)-\left(\cos^2 2x+1\right)=0$$

⑧ سؤال

$$\frac{1-\tan x}{1+\tan x}=\tan^2 x$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$A=\frac{\pi}{4}$ $B=x$

$$\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$$

$$\Rightarrow \tan\left(\frac{\pi}{4}-x\right)=\tan^2 x$$

$$\Rightarrow \frac{\pi}{4}-x=2x+k\pi$$

$$\Rightarrow \frac{\pi}{4}=4x+k\pi \Rightarrow 4x=\frac{\pi}{4}-k\pi$$

$$\Rightarrow \boxed{x=\frac{\pi}{16}+\frac{k\pi}{4}}$$

⑨ سؤال

$$\sqrt{1+\sin 2x}=\sqrt{2}\cos 2x \qquad [0,\pi]$$

بالتربيع

$$\Rightarrow 1+\sin 2x=2\cos^2 2x = 1+\sin 2x-2+2\sin^2 2x=0 \Rightarrow 2\sin^2 2x+\sin 2x-1=0$$

$$2(1-\sin^2 2x)$$

$$2x^2+x-1=0 \Rightarrow \frac{-1\pm\sqrt{1+8}}{4}$$

$$x=\frac{1}{2}\Rightarrow \sin 2x=\frac{1}{2}\Rightarrow 2x=\frac{\pi}{4},\frac{5\pi}{4}$$

$$x=-1\Rightarrow \sin 2x=-1\Rightarrow 2x=X$$

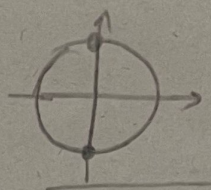
$$\Rightarrow \boxed{x=\frac{\pi}{12},\frac{5\pi}{12}}$$

مرفوض

⑩ سؤال

$[0,2\pi]$

ا) $\sin^2 x-\cos^2 x=1$



$$\begin{cases} x=\frac{\pi}{2}\Rightarrow x=2k\pi+\frac{\pi}{2} \\ x=-\frac{\pi}{2}\Rightarrow x=2k\pi-\frac{\pi}{2} \end{cases} \Rightarrow \boxed{x=k\pi+\frac{\pi}{2}} \Rightarrow [0,2\pi] \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases}$$

ب) $\sin^2 x-\cos^2 x=-1$

$$\begin{cases} x=0\longrightarrow x=2k\pi \\ x=-\frac{\pi}{2}\longrightarrow x=2k\pi-\frac{\pi}{2} \end{cases} \Rightarrow [0,2\pi] \begin{cases} 0 \\ \frac{3\pi}{2} \end{cases}$$