پرنس زندی

۱۴٫۷۵

سؤال ۱

الف) $\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow -2\sin^2 x - 3\sin x + 2 = 0 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

اگر $\sin x = t \Rightarrow 2t^2 + 3t - 2 = 0 \Rightarrow \dfrac{-3 \pm \sqrt{9+16}}{4}$ $t = \frac{1}{2}$ ✓ , $t = -2$ ✗ غ.ق.ق

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$\Rightarrow \sin x = \frac{1}{2} \Rightarrow \{x \mid x = 2k\pi + \frac{\pi}{6} , x = 2k\pi + \frac{5\pi}{6} , k \in \mathbb{Z}\}$ $-1 \leq \sin m \leq 1$

ب) $\cos 2x + \cos x - 2 = 0$

$\cos 2x = 2\cos^2 x - 1 \Rightarrow 2\cos^2 x - 1 + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0$

$\cos x = t \Rightarrow 2t^2 + t - 3 = 0 \Rightarrow \dfrac{-1 \pm \sqrt{1+24}}{4} \Rightarrow$ $t = -\frac{3}{2}$ ✗ غ.ق.ق , $t = 1$ ✓

$-1 \leq \cos x \leq 1$

$\cos x = 1 \Rightarrow \boxed{x = 2k\pi}$

سؤال ۲

الف) $\sin^4 x - \cos^4 x = \sin 4x$ $\Rightarrow -\cos 2x = \sin 4x$

$\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x}(\cancel{\sin^2 x + \cos^2 x}) = \sin 4x$

$\sin 4x = 2\sin 2x \cos 2x$

$\Rightarrow 2\sin 2x \cos 2x + \cos 2x = 0$

$\cos 2x (2\sin 2x + 1) = 0$

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① $\Rightarrow \cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

② $\Rightarrow 2\sin 2x + 1 = 0 \Rightarrow \sin 2x = -\frac{1}{2} \Rightarrow \begin{cases} 2x = 2k\pi - \frac{\pi}{6} \Rightarrow \boxed{x = k\pi - \frac{\pi}{12}} \\ 2x = 2k\pi + \frac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \frac{7\pi}{12}} \end{cases}$

$\Rightarrow \{ x = \frac{k\pi}{2} + \frac{\pi}{4} , x = k\pi - \frac{\pi}{12} , x = k\pi + \frac{7\pi}{12} \mid k \in \mathbb{Z}\}$

ب) $2\cos^2 x + 2\sin x \cos x = 1 \Rightarrow 2(\cos^2 x + \sin x \cos x) = 1 \Rightarrow$

$\cos^2 x + \sin x \cos x = \frac{1}{2} \Rightarrow 1 - \sin^2 x + \sin x \cos x = \frac{1}{2}$

طبق فرمول $\cos^2 x = \dfrac{1 + \cos 2x}{2}$, $\sin x \cos x = \frac{1}{2}\sin 2x$

$\dfrac{1 + \cos 2x}{2} + \frac{1}{2}\sin 2x = \frac{1}{2} \Rightarrow 1 + \cos 2x + \sin 2x = 1 \Rightarrow \cos 2x + \sin 2x = 0$

$\sin 2x = -\cos 2x \xrightarrow{\div \cos 2x} \tan 2x = -1 \Rightarrow 2x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

سوال ۳

۱,۷۵

الف) $\cot x(2\cos x+1)=\frac{1}{\sin x}$

$\frac{\cos x}{\cancel{\sin x}}\times(2\cos x+1)=\frac{1}{\cancel{\sin x}}\Rightarrow 2\cos^2 x+\cos x-1=0$

$\cos x=t\Rightarrow 2x^2+x-1=0\Rightarrow \frac{-1\pm\sqrt{1+8}}{4}=\begin{cases}x=\frac{1}{2}\\ x=-1\end{cases}\Rightarrow \cos x=-1,\frac{1}{2}$

$\Rightarrow \boxed{\begin{array}{l}x=2k\pi\pm\frac{\pi}{3}\\ x=2k\pi+\pi\end{array}}$ ×

اگر $\cos x=-1$ باشد $\sin x=0$ است و عبارت تعریف نشده می‌شود

پس جواب غیر قابل قبول محسوب می‌شود

ب) $2\sin^2 x-\sin x\cos x+\cos^2 x=2$ $\quad a\sin^2 x+b\cos^2 x+c\sin x\cos x=d$

$a \quad c \quad b \quad d \qquad a=d$

$\div\sin^2 x$

$\Rightarrow 2+\cot^2 x-\cot x=2(1+\cot^2 x)\Rightarrow \cot^2 x+\cot x=0\Rightarrow$

$\cot x(\cot x+1)=0$ $\begin{cases}\cot x=0\longrightarrow x=k\pi+\frac{\pi}{2}\\ \cot x=-1\longrightarrow x=k\pi-\frac{\pi}{4}\end{cases}$

سوال ۴

۱,۲۵

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4}$

$\cos A\cos B=\frac{1}{2}(\cos(A+B)+\cos(A-B))=\frac{1}{2}(\underbrace{\cos\frac{\pi}{2}}_{0}+\cos 2x)$

$A=\frac{\pi}{4}+x \qquad B=x-\frac{\pi}{4}$ $\begin{cases}A+B=2x\\ A-B=\frac{\pi}{2}\end{cases}$ $=\frac{1}{2}\cos 2x=\frac{1}{4}\Rightarrow \cos 2x=\frac{1}{2}$

$\Rightarrow 2x=2k\pi\pm\frac{\pi}{3}\Rightarrow \boxed{x=k\pi\pm\frac{\pi}{6}}$

ب) $\cos(2x-\frac{\pi}{9})=-\sin 2x\overset{\mathrm{I}}{\Rightarrow}\cos(2x-\frac{\pi}{9})=\cos(2x+\frac{\pi}{2})$

$-\sin 2x=\cos(\frac{\pi}{2}+2x)$ (I)

$\cos A=\cos B\rightarrow$ ادامه؟

$\cos(2x-\frac{\pi}{9})=-\sin 2x$

$\cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$

$2x-\frac{\pi}{9}=2k\pi\pm(\frac{\pi}{2}+2x)$

+ حالت $\rightarrow 2k\pi+\frac{\pi}{9}+\frac{\pi}{2}=0$ ×

− حالت $\rightarrow 2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x$

$4x=2k\pi-\frac{7\pi}{18}\rightarrow \boxed{x=\frac{k\pi}{2}-\frac{7\pi}{72}}$

سوال ۵

$$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \Rightarrow \frac{\sin 4x}{\sin 2x}+\frac{\cancel{\sin 2x}}{\cancel{\sin 2x}}=$$

$$\frac{2\cancel{\sin 2x}\cos 2x}{\cancel{\sin 2x}}+1=\boxed{2\cos 2x+1}$$

$$2\cos 2x+1-\sin^2 x=\cos^2 x \Rightarrow 2\cos 2x+\cancel{1}=\cancel{\cos^2 x+\sin^2 x} \Rightarrow \cos 2x=0$$

$$\Rightarrow 2x=k\pi+\frac{\pi}{2} \Rightarrow \boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}} \quad k\in\mathbb{Z}$$

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سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2} \qquad \left[-\pi,\frac{3\pi}{2}\right]$$

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$$\frac{1}{\sin\frac{x}{2}}+\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow$$

$$\sin^2\frac{x}{2}=\left(1+\cos\frac{x}{2}\right)^2 \Rightarrow 1+\cos\frac{x}{2}=\pm\sin\frac{x}{2} \quad \begin{cases} 1+\cos\frac{x}{2}=\sin\frac{x}{2} \\ 1+\cos\frac{x}{2}=-\sin\frac{x}{2} \end{cases}$$

$$\Rightarrow \sin\frac{x}{2}-\cos\frac{x}{2}=1$$

$$\Rightarrow \begin{cases} x=2k\pi+\frac{\pi}{2} \\ x=2k\pi+\pi \end{cases}$$

$$\cos\frac{x}{2}+\sin\frac{x}{2}=-1$$

$$\Rightarrow \begin{cases} x=2k\pi-\frac{\pi}{2} \\ x=2k\pi+\pi \end{cases}$$

سوال ۷

$\cos^2 x-\sin^2 x\cos 4x=1$ در بازه $[0,2\pi]$

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$$\frac{1+\cos 2x}{2}-\frac{1-\cos 2x}{2}\times\cos 4x=1$$

$$1+\cos 2x-(1-\cos 2x)\cos 4x=2 \Rightarrow \cos 2x-\cos 4x+\cos 2x\cos 4x=1$$

$$\cos 2x(\cos 4x+1)-(\cos 4x+1)=0$$ ادامه؟

$$\cos^2 x-\sin^2 x\cos 4x=1 \xrightarrow{\cos^2 x=1-\sin^2 x}$$

$$\cancel{1}-\sin^2 x-\sin^2 x\cos 4x=\cancel{1} \longrightarrow \sin^2 x(1+\cos 4x)=0 \quad \begin{cases}\sin x=0\\ \cos 4x=-1\end{cases}$$

$$\sin x=0 \longrightarrow x=k\pi \longrightarrow x=\overset{3}{0},\ \overset{2}{\pi},\ \overset{1}{2\pi}$$

$$\cos 4x=-1 \longrightarrow 4x=2k\pi+\pi \longrightarrow x=\frac{2k\pi+\pi}{4} \longrightarrow x=\overset{5}{\frac{\pi}{4}},\ 3\pi,\ \overset{4}{\frac{5\pi}{4}}$$

معادله ۵ جواب دارد

سؤال ۸

$$\frac{1-\tan x}{1+\tan x}=\tan 3x$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

$A=\frac{\pi}{4}$ $B=x$

$$\tan\left(\frac{\pi}{4}-x\right)=\frac{1-\tan x}{1+\tan x}$$

$\Rightarrow \tan\left(\frac{\pi}{4}-x\right)=\tan 3x$

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$\Rightarrow \frac{\pi}{4}-x=3x+k\pi$

$\Rightarrow \frac{\pi}{4}=4x+k\pi \Rightarrow 4x=\frac{\pi}{4}-k\pi$

$\Rightarrow \boxed{x=\frac{\pi}{16}+\frac{k\pi}{4}}$

سؤال ۹

$$\sqrt{1+\sin 2x}=\sqrt{2}\cos 2x \qquad [0,\pi]$$

$\xrightarrow{\text{توان ۲}} 1+\sin 2x=\underbrace{2\cos^2 2x}_{2(1-\sin^2 2x)} = 1+\sin 2x-2+2\sin^2 2x=0 \Rightarrow 2\sin^2 2x+\sin 2x-1=0$

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$2x^2+x-1=0 \Rightarrow \frac{-1\pm\sqrt{1+8}}{4}$

$x=\frac{1}{2} \Rightarrow \sin 2x=\frac{1}{2} \Rightarrow 2x=\frac{\pi}{6},\ \frac{5\pi}{6}$

$x=-1 \Rightarrow \sin 2x=-1 \Rightarrow 2x=\times$

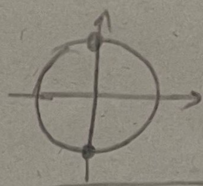
$\times$

$\Rightarrow \boxed{x=\frac{\pi}{12},\ \frac{5\pi}{12}}$ ۲ جواب

به ازای $x=\frac{5\pi}{12}$ ، $\cos 2x$ منفی می‌شود، پس این جواب را نباید لحاظ کرد

سؤال ۱۰ $[0,2\pi]$

الف) $\sin^4 x-\cos^4 x=1$



$\begin{cases} x=\frac{\pi}{2} \Rightarrow x=2k\pi+\frac{\pi}{2} \\ x=-\frac{\pi}{2} \Rightarrow x=2k\pi-\frac{\pi}{2} \end{cases} \Rightarrow \boxed{x=k\pi+\frac{\pi}{2}} \Rightarrow [0,2\pi] \ \frac{\pi}{2},\ \frac{3\pi}{2}$

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ب) $\sin^3 x-\cos^3 x=-1$

$\begin{cases} x=0 \rightarrow x=2k\pi \\ x=-\frac{\pi}{2} \rightarrow x=2k\pi-\frac{\pi}{2} \end{cases} \Rightarrow [0,2\pi] \ 0$ و 2π ، $\frac{3\pi}{2}$

سوال ۶

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \boxed{\tan\frac{x}{4}}$$

$$\frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} = \frac{1}{\tan\frac{x}{4}} = \boxed{\cot\frac{x}{4}}$$

$$\tan\frac{x}{4} = \cot\frac{x}{4} \rightarrow \tan\frac{x}{4} = \tan\left(\frac{\pi}{2} - \frac{x}{4}\right)$$

$$\frac{x}{4} = k\pi + \frac{\pi}{2} - \frac{x}{4} \rightarrow \frac{x}{2} = k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \pi$$

جواب ها: $k=0 \rightarrow x=\pi$

$k=-1 \rightarrow x=-\pi \rightarrow$ اختلاف $= |\pi - (-\pi)| = 2\pi$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0$$

$\sin 2x = -1$

$\sin 2x = +\frac{1}{2}$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0\le x\le \pi} x = \frac{3\pi}{4}$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0\le x\le \pi} x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12}$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، چون جواب رادیکال است غ ق ق است

★ معادله ۲ جواب دارد

سوال ۱۰

در سوالات به فرم $\pm\sin^n x \pm \cos^n x$ برابر 1 یا -1 . فقط کافی است نقاط اصلی را چک کنی !

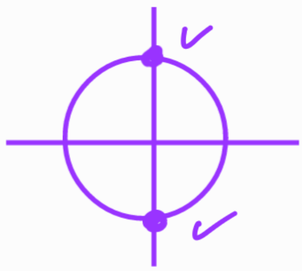
ب)

$\sin^3 x - \cos^2 x = 1$

$x = 2\pi$ یا 0

$x = \frac{3\pi}{2}$

الف)



$\sin^4 x - \cos^4 x = 1$

$x = \frac{\pi}{2}$

$x = \frac{3\pi}{2}$