

Date:

Sub:

(14)

باران نسایی دوازدهم دخترب

۱۸، ۵

$$\text{الف) } \cos^2 x = \sqrt{2} \sin x - 1 \quad -1$$

$$\sin^2 x = \frac{1 - \cos^2 x}{\sqrt{2}} \rightarrow \sqrt{2} \sin^2 x = 1 - \cos^2 x \rightarrow 1 - \sqrt{2} \sin^2 x = \cos^2 x \quad 2$$

$$\sqrt{2} \sin^2 x + \sqrt{2} \sin x - \sqrt{2} = 0 \xrightarrow{\text{توی}} t^2 + \sqrt{2}t - \sqrt{2} = 0$$

$$(t + \sqrt{2})(t - 1) = 0 \quad \left[\begin{array}{l} \sin x = -\sqrt{2} \\ \sin x = \frac{1}{\sqrt{2}} \end{array} \right. \checkmark$$

$$\sin x = \sin \frac{\pi}{4} \quad \checkmark$$

$$x = 2K\pi + \frac{\pi}{4}$$

$$x = 2K\pi + \pi - \frac{\pi}{4} \rightarrow x = 2K\pi + \frac{3\pi}{4} \quad \checkmark$$

$$\text{ب) } \cos^2 x + \cos x - \sqrt{2} = 0$$

$$\cos^2 x = \frac{1 + \cos^2 x}{\sqrt{2}} \rightarrow \cos^2 x = \sqrt{2} \cos^2 x - 1$$

$$\sqrt{2} \cos^2 x + \cos x - \sqrt{2} = 0 \quad \left[\begin{array}{l} \cos x = 1 \quad \checkmark \\ \cos x = \frac{-\sqrt{2}}{\sqrt{2}} \quad \times \end{array} \right.$$

$$\cos x = \cos \pi$$

$$x = 2K\pi \quad \checkmark$$

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$$\text{الف) } \sin^E x - \cos^E x = \sin^E x$$

-P

$$\underbrace{(\sin^P x + \cos^P x)}_1 \underbrace{(\sin^P x - \cos^P x)}_{-\cos^P x} = \sin^E x$$

$$-\sin\left(\frac{\pi}{P} - Px\right) = \sin^E x \rightarrow \sin^E x = \sin\left(Px - \frac{\pi}{P}\right)$$

P

$$Px = PK\pi + Px - \frac{\pi}{P} \rightarrow Px = PK\pi - \frac{\pi}{P} \rightarrow x = \frac{PK\pi}{P} - \frac{\pi}{P}$$

$$Ex = PK\pi + \pi - Px + \frac{\pi}{P} \rightarrow Ex = PK\pi + \frac{P\pi}{P} \rightarrow x = \frac{PK\pi}{P} + \frac{\pi}{P}$$

$$\text{ب) } P\cos^P x + P\sin^P x \cos^P x = 1$$

$$P\sin^P x \cos^P x = \underbrace{1 - \cos^P x}_{\sin^P x} - \cos^P x$$

$$\rightarrow \sin^P x = -\cos^P x \xrightarrow{\div \cos^P x} \tan^P x = -1$$

$$Px = K\pi - \frac{\pi}{P} \rightarrow x = \frac{K\pi}{P} - \frac{\pi}{P}$$

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غیر قابل قبول (سینوس x نباید صفر شود)

$$\text{الف)} \quad \cot x (2 \cos x + 1) = \frac{1}{\sin x}$$

$$\frac{\cos x}{\cancel{\sin x}} (2 \cos x + 1) = \frac{1}{\cancel{\sin x}} \rightarrow 2 \cos^2 x + \cos x - 1 = 0$$

$\sin x \neq 0$

$$\cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$\cos x = \cos \frac{\pi}{2} \rightarrow x = 2K\pi \pm \frac{\pi}{2} \quad \checkmark$$

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$$\text{ب)} \quad \cancel{2 \sin^2 x} - \sin x \cos x + \cos^2 x = 2$$

$$\downarrow \quad \cancel{2 \sin^2 x} + 2 \cos^2 x$$

$$-\sin x \cos x - \cos^2 x = 0$$

$$-\cos x (\sin x + \cos x) = 0$$

$$\sin x = -\cos x$$

$$\xrightarrow{\div \cos x} \tan x = -1$$

$$x = K\pi - \frac{\pi}{4} \quad \checkmark$$

$$\cos x = 0 \rightarrow x = 2K\pi + \frac{\pi}{2}$$

$$\tan x = -1 \Rightarrow x = K\pi - \frac{\pi}{4}$$

PITICO

$$\text{ا) } \cos\left(\frac{\pi}{r} + x\right) \cos\left(x - \frac{\pi}{r}\right) = \frac{1}{r}$$

(1,20) -r

$$\cos\left(\frac{\pi}{r} + x\right) = \sin\left(\frac{\pi}{r} - x\right) = -\sin\left(x - \frac{\pi}{r}\right)$$

$$\frac{1}{r} \sin\left(x - \frac{\pi}{r}\right) = \frac{-1}{r} \rightarrow \sin\left(x - \frac{\pi}{r}\right) = \frac{-1}{r}$$

$$-\cos rx = \frac{-1}{r} \rightarrow \cos rx = \frac{1}{r}$$

$$rx = rK\pi + \frac{\pi}{r} \rightarrow x = K\pi + \frac{\pi}{r} \quad \checkmark$$

$$\text{ب) } \cos\left(rx - \frac{\pi}{a}\right) = -\sin rx$$

$$\cos rx \cos \frac{\pi}{a} + \sin rx \sin \frac{\pi}{a} = -\sin rx$$

$$\frac{\div \cos rx}{\cos rx \neq 0} \cos \frac{\pi}{a} + \tan rx \sin \frac{\pi}{a} = -\tan rx$$

$$\cos \frac{\pi}{a} = -\tan rx \sin \frac{\pi}{a} - \tan rx$$

$$-\tan rx (\sin \frac{\pi}{a} + 1)$$

$$-\tan rx = \frac{\cos \frac{\pi}{a}}{\sin \frac{\pi}{a} + 1} \xrightarrow{\arctan} rx = \arctan\left(\frac{-\cos \frac{\pi}{a}}{\sin \frac{\pi}{a} + 1}\right) + K\pi$$

$$\rightarrow x = \frac{1}{r} \arctan\left(\frac{-\cos \frac{\pi}{a}}{\sin \frac{\pi}{a} + 1}\right) + \frac{K\pi}{r}$$

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$$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \quad - \text{a}$$

$$\frac{\sin 4x + \sin 2x}{\sin 2x} = 1$$

$$\rightarrow \frac{\sin 4x + \sin 2x}{\sin 2x} - 1 = 0$$

1/8

$$\frac{\sin 4x}{\sin 2x} = 0 \rightarrow \sin 4x = 0$$
$$\sin 2x \neq 0$$

به ازای بعضی مقادیر k مقدم صفر می شود!

$$4x = k\pi \rightarrow x = \frac{k\pi}{4}$$

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$$\frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1}{\sin \frac{x}{r}} + \cot \frac{x}{r} \quad - 9$$

$$\frac{\sin \frac{x}{r}}{1 + \cos \frac{x}{r}} = \frac{1 + \cos \frac{x}{r}}{\sin \frac{x}{r}} \quad \sin \frac{x}{r} \neq 0$$

$$1 + \cos \frac{x}{r} \neq 0$$

$$\sin^2 \frac{x}{r} = (1 + \cos \frac{x}{r})^2$$

$$\sin^2 \frac{x}{r} = \underbrace{\cos^2 \frac{x}{r} + 2 \cos \frac{x}{r} + 1}_{1 - \sin^2 \frac{x}{r}} \rightarrow \sin^2 \frac{x}{r} = 1 + 2 \cos \frac{x}{r} - \sin^2 \frac{x}{r}$$

$$r \sin^2 \frac{x}{r} = r(1 + \cos \frac{x}{r}) \rightarrow \sin^2 \frac{x}{r} = 1 + \cos \frac{x}{r}$$

$$1 - \cos^2 \frac{x}{r} = 1 + \cos \frac{x}{r}$$

$$\cos^2 \frac{x}{r} + \cos \frac{x}{r} = 0 \rightarrow \cos \frac{x}{r} (1 + \cos \frac{x}{r}) = 0$$

$$[-1, \frac{3\pi}{r}] \rightarrow \cos \frac{x}{r} = 0 \rightarrow \frac{x}{r} = \frac{\pi}{r}, \frac{3\pi}{r}$$

$$\cos \frac{x}{r} = -1 \rightarrow \text{غير قابل قبول} \quad \sin x \neq 0$$

$$x = -\pi, \pi, \frac{3\pi}{r}$$

$$\text{و باز } \rightarrow [-\pi, \frac{3\pi}{r}] \Rightarrow |\pi - (-\pi)| = 2\pi$$

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$$\cos^2 x - \sin^2 x \cos^2 x = 1 \quad -V \quad \text{تعداد جواب در فاصله } [0, 2\pi]$$

$$-\sin^2 x \cos^2 x = 1 - \cos^2 x$$

$$-\cancel{\sin^2 x} \cos^2 x = \cancel{\sin^2 x}$$

$$\text{(I)} \\ \text{if } \sin x = 0 \rightarrow x = k\pi$$

$$-\cos^2 x = 1$$

Ⓟ

$$\rightarrow \cos^2 x = -1 \rightarrow x = 2k\pi \pm \pi$$

$$x = \frac{2k\pi}{2} \pm \frac{\pi}{2} \rightarrow x \in \left\{ k\pi, \frac{2k\pi}{2} \pm \frac{\pi}{2} \right\}$$

$$\text{جواب } \leftarrow \frac{5\pi}{2}, \pi, 2\pi, \pi, \frac{\pi}{2} \leftarrow [0, 2\pi] \text{ در این}$$

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$$\frac{1 - \tan x}{1 + \tan x} = \tan^2 x \quad - 1$$

$$\frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} = \left(\frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x + \sin x}{\cos x}} \right) = \frac{\cos x - \sin x}{\cos x + \sin x}$$

$$\frac{\cos x - \sin x}{\cos x + \sin x} = \frac{\sin^2 x}{\cos^2 x}$$

$$\cos^2 x \cos x - \sin x \cos^2 x = \cos x \sin^2 x + \sin x \sin^2 x$$

$$\cos^2 x \cos x - \sin x \sin^2 x = \sin x \cos^2 x + \cos x \sin^2 x$$

$$\cos^2 x = \sin^2 x \xrightarrow{\div \cos^2 x} \tan^2 x = 1$$

$$x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{1} + \frac{\pi}{1}$$

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$$\sqrt{1 + \sin^2 x} = \sqrt{2} \cos^2 x \quad \text{جواب } [0, \pi] \quad \text{--- 9}$$

$$1 + \sin^2 x = 2 \cos^2 x$$

$$\sin^2 x = 2 \cos^2 x - 1$$

$$\rightarrow \sin^2 x = \cos^2 x \quad \rightarrow \sin^2 x = 1 - \sin^2 x$$

$$\sin^2 x = t \rightarrow 2t^2 + t - 1 = 0 \quad \begin{cases} t = -1 \\ t = \frac{1}{2} \end{cases}$$

$$\sin^2 x = -1 \quad \text{---}$$

$$\rightarrow x = k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{2}$$

دستاره شده $\frac{\pi}{2}$ غیر قابل قبول جواب است

$$\cos^2 x \geq 0 \rightarrow \cos^2 x \rightarrow n = \frac{\pi}{2} = \cos^2 \frac{\pi}{2} \dots \checkmark$$

$$\sin^2 x = \frac{1}{2} \rightarrow x = k\pi + \frac{\pi}{4}$$

$$x = k\pi + \frac{\pi}{4} \rightarrow \frac{\pi}{4}$$

$$x = k\pi + \pi - \frac{\pi}{4} \rightarrow k\pi + \frac{3\pi}{4} = x$$

$$\frac{3\pi}{4} \leftarrow \text{دستاره شده} \quad \frac{3\pi}{4} \leftarrow \text{قبل قبول است}$$

$$\cos \frac{10\pi}{12} < 0 \quad \frac{\pi}{12} \leftarrow \text{جواب قدر}$$

$$n = \frac{\pi}{2} \text{ و } \frac{\pi}{4}$$

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$$(1) \sin^2 x - \cos^2 x = 1$$

$$[0, 2\pi] \quad -10$$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1$$

$\underbrace{\hspace{1.5cm}} \quad \underbrace{\hspace{1.5cm}}$
1 $-\cos^2 x$

$$-\cos^2 x = 1 \rightarrow \cos^2 x = -1$$

2

$$x = k\pi \pm \pi \quad \checkmark$$

دائیں بازو کے جواب طرد سے π

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$$\text{ب) } \sin^3 x - \cos^3 x = -1$$

$$(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)$$

$$= (\sin x - \cos x)(1 + \sin x \cos x)$$

$$x = \pi/2 = 0/2 = \frac{\pi}{2} \text{ لأن } \sin \frac{\pi}{2} = 1$$

$$\frac{\sin x \cos x + \sin x}{\sin x} = \sin x + \cos x$$

سوال ٦

$$\frac{\sin x (\cos x + 1)}{\sin x} = 1 \rightarrow \cos x + 1 = 1 \rightarrow \cos x = 0$$

$$x = k\pi + \frac{\pi}{2} \rightarrow x = k\pi + \frac{\pi}{2}$$

$$\cos(x - \frac{\pi}{4}) = -\sin x$$

سوال ٧

$$\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} + x)$$

$$x - \frac{\pi}{4} = 2k\pi \pm (\frac{\pi}{4} + x) \rightarrow 2k\pi + \frac{\pi}{4} + \frac{\pi}{4} = 0 \quad \times$$

$$\rightarrow x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} - x$$

$$x = 2k\pi - \frac{\pi}{4} \rightarrow x = \frac{8k\pi - \pi}{4} = \frac{8k\pi - \pi}{4}$$

سوال ۹

$$\sqrt{1 + \sin 2x} = \sqrt{2} \cos x \xrightarrow{\text{توان ۲}} 1 + \sin 2x = 2 \cos^2 x$$

$$1 + \sin 2x = 2 - 2 \sin^2 x \rightarrow 2 \sin^2 x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[0 \leq x \leq \pi]{k \in \mathbb{Z}} x = \frac{3\pi}{4}$$

$$\hookrightarrow \sin 2x = \frac{1}{2}$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \text{ یا } 2k\pi + \frac{5\pi}{6} \xrightarrow[0 \leq x \leq \pi]{k \in \mathbb{Z}} x = \frac{\pi}{12} \text{ یا } \frac{5\pi}{12}$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می کنند
 به ازای $\sin 2x = \frac{5\pi}{12}$ منفر مرشود چون جواب را ایست نمی دز است

۲ معادله ۲ جواب دارد