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الف

$$\cos 2n = 3 \sin n - 1$$

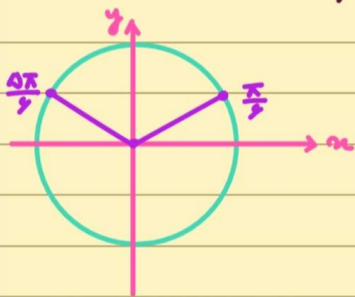
$$1 - 2 \sin^2 n = 3 \sin n - 1 \rightarrow 2 \sin^2 n + 3 \sin n - 2 = 0 \rightarrow \Delta = 9 - 4(-2) = 17$$

$$\sin n = \frac{-3 \pm \sqrt{17}}{4}$$

$\rightarrow \sin n = -2$ ~~ووجد~~
 $\rightarrow \sin n = \frac{1}{2}$ ✓

$$\sin n = \frac{1}{2} \rightarrow n = 2k\pi + \frac{\pi}{6}$$

$$\rightarrow n = 2h\pi + \pi - \frac{\pi}{6}$$



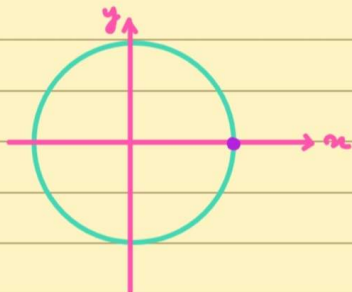
ب.

$$\cos 2n + \cos n - 1 = 0$$

$$2 \cos^2 n - 1 + \cos n - 1 = 0 \rightarrow 2 \cos^2 n + \cos n - 2 = 0 \xrightarrow{a+b+c=0} \cos n$$

$\rightarrow \frac{c}{a} = -\frac{2}{2} = -1$
 $\rightarrow -1$ ✓

$$\cos n = 1 \rightarrow n = 2k\pi$$



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الف

$$\cancel{\sin^2 m} - \cancel{\cos^2 m} = \cancel{\sin^2 m}$$

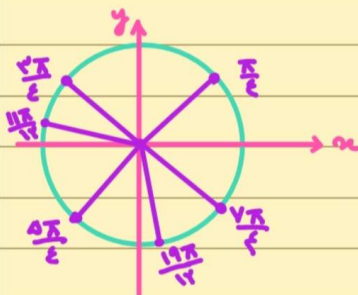
$$2 \sin^2 m \cos m + \cos m = 0 \rightarrow \cos m (2 \sin^2 m + 1) = 0$$

$\cos m = 0$
 $2 \sin^2 m + 1 = 0$

$$\cos m = 0 \rightarrow m = 2k\pi \pm \frac{\pi}{2} \rightarrow m = k\pi \pm \frac{\pi}{2} \quad \textcircled{\text{I}}$$

$$2 \sin^2 m + 1 = 0 \rightarrow \sin^2 m = -\frac{1}{2} \rightarrow m = 2k\pi - \frac{\pi}{2} = k\pi - \frac{\pi}{2} \quad \textcircled{\text{II}}$$

$$m = 2k\pi + \pi + \frac{\pi}{2} = \frac{5}{2}k\pi + \frac{\pi}{2} \quad \textcircled{\text{III}}$$



ب

$$2 \cos^2 m + 2 \sin^2 m \cos m = 1$$

$$\cancel{2 \cos^2 m} - 1 + \cancel{2 \sin^2 m} \cos m = 0$$

$\cos m$ $\sin^2 m$

$$\cos^2 m + \sin^2 m = 0 \rightarrow (\cos^2 m + \sin^2 m) = \cancel{\cos^2 m} + \sin^2 m + \cancel{2 \sin^2 m \cos m}$$

$$\sin^2 m = -1 \rightarrow m = 2k\pi + \frac{3\pi}{2} \rightarrow m = \frac{k\pi}{2} + \frac{3\pi}{2}$$

$$\cot x (2 \cos x + 1) = \frac{1}{\sin x}, \quad \sin x \neq 0$$

$$\frac{\cos x}{\sin x} \rightarrow \cos x (2 \cos x + 1) = 1 \rightarrow 2 \cos^2 x + \cos x - 1 = 0$$

$$a + c = b \rightarrow \cos x \begin{cases} \frac{-c}{a} = \frac{1}{2} \\ -1 \end{cases}$$

$$\cos x = \frac{1}{2} \rightarrow x = 2k\pi \pm \frac{\pi}{3}$$

$$\cos x = -1 \rightarrow x = 2k\pi + \pi$$

$$2 \sin^2 x - \sin x \cos x + \cos^2 x = 2$$

$$\sin^2 x + \cos^2 x = 1 \rightarrow -1 + \sin^2 x + \sin x \cos x = 0$$

$$\div \cos^2 x \rightarrow \tan^2 x + \tan x - \frac{1}{\cos^2 x} = -\tan x - 1 = 0$$

(cos x ≠ 0 أو 1) -(1 + tan^2 x)

$$\tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4} \quad \star$$

$$\begin{matrix} \text{عوضه} \\ \cos x = 0 \\ \text{بالمقام} \end{matrix} \rightarrow 2 \sin^2 x = 2 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1$$

$$x = k\pi + \frac{\pi}{2} \quad \star$$

$$\cos\left(\frac{\pi}{\epsilon} + n\right) \cos\left(\frac{\pi}{\epsilon} - n\right) = \frac{1}{\epsilon} \rightarrow n + \frac{\pi}{\epsilon} + \frac{\pi}{\epsilon} - n = \frac{\pi}{\epsilon} \quad \text{الف)}$$

$$\cos\left(n + \frac{\pi}{\epsilon}\right) \sin\left(n + \frac{\pi}{\epsilon}\right) = \frac{1}{\epsilon} \sin\left(2n + \frac{\pi}{\epsilon}\right) = \frac{1}{\epsilon}$$

$$\cos 2n = \frac{1}{\epsilon}$$

$$2n = 2k\pi \pm \frac{\pi}{\epsilon} \rightarrow n = k\pi \pm \frac{\pi}{\epsilon}$$

$$\cos\left(2n - \frac{\pi}{\epsilon}\right) = -\sin 2n \quad \text{ب)}$$

$$\cos\left(2n - \frac{\pi}{\epsilon}\right) = \cos\left(\frac{\pi}{\epsilon} - 2n\right) \rightarrow \cos\left(2n - \frac{\pi}{\epsilon}\right) + \cos\left(\frac{\pi}{\epsilon} - 2n\right) = 0$$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos\left(2n - \frac{\pi}{\epsilon}\right) + \cos\left(\frac{\pi}{\epsilon} - 2n\right) = 2 \cos\left(\frac{\pi}{\epsilon}\right) \cos\left(2n - \frac{11\pi}{\epsilon}\right) = 0$$

$$\cos \frac{\pi}{\epsilon} \neq 0, \quad \cos\left(2n - \frac{11\pi}{\epsilon}\right) = 0$$

$$2n - \frac{11\pi}{\epsilon} = k\pi + \frac{\pi}{\epsilon} \rightarrow 2n = k\pi + \frac{12\pi}{\epsilon}$$

$$n = \frac{k\pi}{2} + \frac{6\pi}{\epsilon}$$

-5

$$\frac{\sin 2n + \sin 4n}{\sin n} - \sin^2 n = \cos^2 n$$

$$\frac{\sin 2n}{\sin n} + 1 = \cancel{\sin^2 n} + \cos^2 n \rightarrow \frac{2 \cancel{\sin n} \cos n}{\cancel{\sin n}} = 0$$

$$2 \cos n = 0 \quad \cos n = 0 \quad 2n = 2k\pi + \frac{\pi}{2} \rightarrow n = k\pi + \frac{\pi}{4}$$

-6

$$\frac{\sin \frac{n}{2}}{1 + \cos \frac{n}{2}} = \frac{1}{\sin \frac{n}{2}} + \cot \frac{n}{2}$$

$$\tan \frac{n}{2} = \frac{1}{\tan \frac{n}{2}} + \cot \frac{n}{2} + \cot \frac{n}{2}$$

$$\tan \frac{n}{2} = t \rightarrow t = \frac{1}{t} + \frac{1}{t} + \frac{1-t^2}{2t}$$

$$t = \frac{t^2+1}{2t} + \frac{1-t^2}{2t} = \frac{1}{t} \implies t = \frac{1}{t} \implies t = \pm 1$$

$$\tan \frac{n}{2} = \pm 1 \rightarrow \frac{n}{2} = k\frac{\pi}{2} + \frac{\pi}{4} \rightarrow n = (2k+1)\pi$$

$$n \in [-\pi, \frac{3\pi}{2}] \rightarrow n_1 = -\pi, n_2 = \pi \rightarrow |\pi - (-\pi)| = 2\pi$$

-7

$$\cos^2 n - \sin^2 n \cos 2n = 1$$

$$-\sin^2 n \cos 2n = 1 - \cancel{\sin^2 n}$$

$$\sin^2 n \neq 0 \rightarrow \cos 2n = -1 \quad 2n = 2k\pi + \pi \rightarrow n = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$\sin^2 n = 0 \rightarrow \sin n = 0 \quad \sin n = 0 \rightarrow n = 2k\pi$$

$$n \in [0, 2\pi] \rightarrow n \in \{0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi\}$$

جواب دارد

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$$\frac{\tan \alpha - 1}{\tan \alpha + 1} = \tan \alpha$$

$$\tan\left(\frac{\pi}{4} - \alpha\right) = \frac{\tan \frac{\pi}{4} - \tan \alpha}{1 + \tan \frac{\pi}{4} \tan \alpha} = \frac{1 - \tan \alpha}{1 + \tan \alpha}$$

$$\tan\left(\frac{\pi}{4} - \alpha\right) = \tan \alpha$$

$$\frac{\pi}{4} - \alpha = \alpha + k\pi \longrightarrow \alpha = \frac{\pi}{4} - k\pi \longrightarrow \alpha = \frac{\pi}{4} - \frac{k\pi}{1}$$

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$$\sqrt{1 + \sin 2\alpha} = \sqrt{2} \cos \alpha$$

$$\sqrt{(\sin^2 \alpha + \cos^2 \alpha) + 2 \sin \alpha \cos \alpha} = \sqrt{2} \cos \alpha$$

$(\sin \alpha + \cos \alpha)^2$

$$|\sin \alpha + \cos \alpha| = \sqrt{2} \cos \alpha$$

$$\sqrt{2} \left| \sin\left(\alpha + \frac{\pi}{4}\right) \right| = \sqrt{2} \cos \alpha$$

$$\sin\left(\alpha + \frac{\pi}{4}\right) \geq 0 \longrightarrow \sin\left(\alpha + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4} - \alpha\right)$$

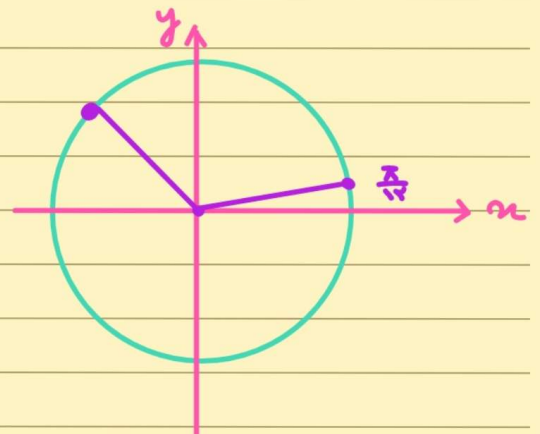
$$\alpha + \frac{\pi}{4} = \frac{\pi}{4} - \alpha + 2k\pi$$

$$2\alpha = 2k\pi \longrightarrow \alpha = k\pi \longrightarrow \left\{ \frac{\pi}{4} \right\}$$

$$\sin\left(\alpha + \frac{\pi}{4}\right) < 0 \longrightarrow \sin\left(\alpha + \frac{\pi}{4}\right) = \sin\left(\alpha - \frac{\pi}{4}\right)$$

$$\alpha + \frac{\pi}{4} = \alpha - \frac{\pi}{4} + 2k\pi$$

$$\alpha = \frac{\pi}{4} - 2k\pi \longrightarrow \left\{ \frac{3\pi}{4} \right\}$$



الاجابة

$$\sin^2 u - \cos^2 u = 1$$

$$\sin^2 u - \cos^2 u = 1$$

$$\cos 2u = -1 \quad 2u = 2k\pi + \pi \quad u = k\pi + \frac{\pi}{2} \quad \xrightarrow{u \in [0, 2\pi]} u \in \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

$$\sin^2 u - \cos^2 u = -1$$

(ب)

بررسی چهار نقطه اصلی:

$$\sin^2 u - \cos^2 u = -1$$

- $u=0 \rightarrow 0 - (1) = -1 \checkmark$
- $u=\frac{\pi}{2} \rightarrow 1 - (0) = 1 \times$
- $u=\pi \rightarrow 0 - (-1) = 1 \times$
- $u=\frac{3\pi}{2} \rightarrow -1 - (0) = -1 \checkmark$