۱۹٫۷۵ آفرین خیلی خوب نوشتی :))

سارینا عابدینی

۱ـ

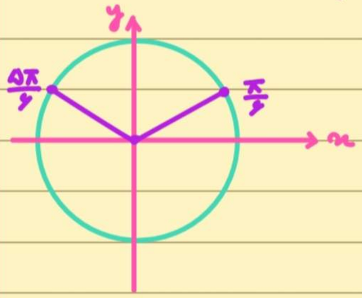
الف) ۲

$\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \longrightarrow 2\sin^2 x + 3\sin x - 2 = 0 \longrightarrow \Delta = 9 - 4(-4) = 25$

$\sin x = \frac{-3 \pm 5}{4}$ → $\sin x = -2$ غ ق ق ; $\sin x = \frac{1}{2}$ ✓

$\sin x = \frac{1}{2} \longrightarrow x = 2k\pi + \frac{\pi}{6}$ ✓ ; $x = 2k\pi + \pi - \frac{\pi}{6}$ ✓

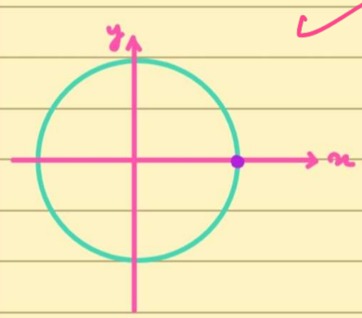


ب)

$\cos 2x + \cos x - 1 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0 \longrightarrow 2\cos^2 x + \cos x - 3 = 0 \xrightarrow{a+b+c=0} \cos x$ → $\frac{c}{a} = \frac{-3}{2}$ غ ق ق ; 1 ✓

$\cos x = 1 \longrightarrow x = 2k\pi$ ✓



سارینا عابدینی

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الف)

$$-\cos 2x \qquad 2\sin 2x\cos 2x$$

$$\cancel{\sin^4 x - \cos^4 x} = \cancel{\sin 4x}$$

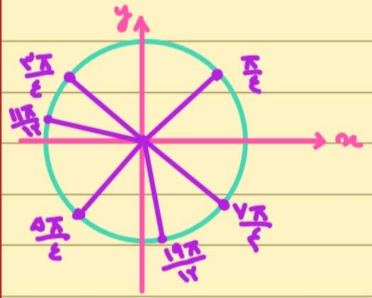
۲

$$2\sin 2x\cos 2x + \cos 2x = 0 \rightarrow \cos 2x(2\sin 2x + 1) = 0 \begin{cases} \cos 2x = 0 \\ 2\sin 2x + 1 = 0 \end{cases}$$

$$\cos 2x = 0 \rightarrow 2x = 2k\pi \pm \frac{\pi}{2} \rightarrow x = k\pi \pm \frac{\pi}{4} \quad \text{(I)}$$

$$2\sin 2x + 1 = 0 \rightarrow \sin 2x = \frac{-1}{2} \rightarrow 2x = 2k\pi - \frac{\pi}{6} = k\pi - \frac{\pi}{12} \quad \text{(II)}$$

$$\rightarrow 2x = 2k\pi + \pi + \frac{\pi}{6} = \cancel{\frac{3}{2}}k\pi + \frac{\pi}{12} \quad \text{(III)}$$



ب)

$$2\cos^2 x + 2\sin x\cos x = 1$$

$$\underbrace{\cancel{2\cos^2 x - 1}}_{\cos 2x} + \underbrace{\cancel{2\sin x\cos x}}_{\sin 2x} = 0$$

$$\cos 2x + \sin 2x = 0 \longrightarrow (\cos 2x + \sin 2x)^2 = \overbrace{\cancel{\cos^2 2x + \sin^2 2x}}^{1} + \overbrace{\cancel{2\sin 2x\cos 2x}}^{\sin 4x}$$

$$\sin 4x = -1 \longrightarrow 4x = 2k\pi + \frac{3\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{8}$$

سارینا عابدینی

۳- ۱٫۷۵

الف)

$\cot x(2\cos x+1)=\frac{1}{\sin x}$, $\sin x\neq 0$

$\xrightarrow{\times\frac{\sin x}{\sin x}} \cos x(2\cos x+1)=1 \longrightarrow 2\cos^2 x+\cos x-1=0$

$\xrightarrow{a+c=b} \cos x \to \frac{-c}{a}=\frac{1}{2}$, $\to -1$

اگر $\cos x=-1$ باشد، $\sin x=0$ است و عبارت تعریف نشده می‌شود
پس جواب غیر قابل قبول محسوب می‌شود

$\cos x=\frac{1}{2} \longrightarrow x=2k\pi\pm\frac{\pi}{3}$

$\cos x=-1 \longrightarrow x=2k\pi+\pi$

ب)

$2\sin^2 x-\sin x\cos x+\cos^2 x=2$

$\xrightarrow{\sin^2 x+\cos^2 x=1} -1+\sin^2 x+\sin x\cos x=0$

$\xrightarrow[(\text{اگر } \cos x\neq 0)]{\div\cos^2 x} \tan^2 x+\tan x-\cancel{\frac{1}{\cos^2 x}}=-\tan x-1=0$

$-(1+\tan^2 x)$

$\tan x=-1 \longrightarrow x=k\pi-\frac{\pi}{4}$ ★

$\xrightarrow[\cos x=0 \text{ را بررسی می‌کنیم}]{\text{حالت خاص}} 2\sin^2 x=2 \longrightarrow \sin^2 x=1 \longrightarrow \sin x=\pm 1$

$x=k\pi+\frac{\pi}{2}$ ★

سارینا عابدینی

۴-

الف)

$$\cos\left(\frac{\pi}{4}+x\right)\overset{\cos\left(\frac{\pi}{4}-x\right)}{\cancel{\cos\left(x-\frac{\pi}{4}\right)}}=\frac{1}{4} \to x+\frac{\pi}{4}+\frac{\pi}{4}-x=\frac{\pi}{2}$$

$$\cos\left(x+\frac{\pi}{4}\right)\sin\left(x+\frac{\pi}{4}\right)=\frac{1}{2}\underset{\cos 2x}{\cancel{\sin\left(2x+\frac{\pi}{2}\right)}}=\frac{1}{4}$$

$$\cos 2x=\frac{1}{2}$$

$$2x=2k\pi\pm\frac{\pi}{3}\longrightarrow x=k\pi\pm\frac{\pi}{6}$$

۲

ب)

$$\cos\left(2x-\frac{\pi}{9}\right)=-\sin 2x$$

$$\cos\left(2x-\frac{\pi}{9}\right)=\cos\left(\frac{\pi}{2}-2x\right)\longrightarrow\cos\left(2x-\frac{\pi}{9}\right)+\cos\left(\frac{\pi}{2}-2x\right)=0$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos\left(2x-\frac{\pi}{9}\right)+\cos\left(\frac{\pi}{2}-2x\right)=2\cos\left(\frac{7\pi}{36}\right)\cos\left(2x-\frac{11\pi}{36}\right)=0$$

$$\cos\frac{7\pi}{36}\neq 0 \quad , \quad \cos\left(2x-\frac{11\pi}{36}\right)=0$$

$$2x-\frac{11\pi}{36}=k\pi+\frac{\pi}{2}\longrightarrow 2x=k\pi+\frac{29\pi}{36}$$

$$x=\frac{k\pi}{2}+\frac{29\pi}{72}$$

سارینا عابدینی

۵-

$$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$$

$$\frac{\sin 4x}{\sin 2x} + 1 = \cancel{\sin^2 x + \cos^2 x}^{\,1} \longrightarrow \frac{2\cancel{\sin 2x}\cos 2x}{\cancel{\sin 2x}} = 0$$

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$$2\cos 2x = 0 \qquad \cos 2x = 0 \qquad 2x = 2k\pi \pm \frac{\pi}{2} \to x = k\pi \pm \frac{\pi}{4}$$

۶-

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \cot\frac{x}{2}$$

$$\tan\frac{x}{4} = \frac{1}{2}\tan\frac{x}{4} + \frac{1}{2}\cot\frac{x}{4} + \cot\frac{x}{2}$$

$$\xrightarrow{\tan\frac{x}{4}=t} t = \frac{1}{2}\left(t+\frac{1}{t}\right) + \frac{1-t^2}{2t}$$

۲

$$t = \frac{t^2+1}{2t} + \frac{1-t^2}{2t} = \frac{1}{t} \Longrightarrow t = \frac{1}{t} \Longrightarrow t = \pm 1$$

$$\tan\frac{x}{4} = \pm 1 \longrightarrow \frac{x}{4} = k\frac{\pi}{2} + \frac{\pi}{4} \longrightarrow x = (2k+1)\pi$$

$$\xrightarrow{x\in[-\pi,\frac{3\pi}{2}]} x_1 = -\pi,\ x_2 = \pi \longrightarrow |\pi-(-\pi)| = 2\pi$$

۷-

$$\cos^2 x - \sin^2 x\cos 3x = 1$$

$$-\sin^2 x\cos 3x = \cancel{1-\cos^2 x}^{\,\sin^2 x}$$

$$\xrightarrow{\sin^2 x\neq 0} \cos 3x = -1 \qquad 3x = 2k\pi + \pi \longrightarrow x = \frac{k\pi}{3} + \frac{\pi}{3}$$

$$\xrightarrow{\sin^2 x=0} \sin x = 0 \qquad \sin x = 0 \longrightarrow x = 2k\pi$$

۲

$$\xrightarrow{x\in[0,2\pi]} x \in \left\{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi\right\}$$

۵ جواب دارد

سارینا عابدینی

۸-

$$\frac{\tan x - 1}{\tan x + 1} = \tan x$$

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \frac{1 - \tan x}{1 + \tan x}$$

۲

$$\tan\left(\frac{\pi}{4} - x\right) = \tan 3x$$

$$\frac{\pi}{4} - x = 3x + k\pi \longrightarrow 4x = \frac{\pi}{4} - k\pi \longrightarrow x = \frac{\pi}{16} - \frac{k\pi}{4}$$

۹-

$$\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x$$

$$\sqrt{\underbrace{(\sin^2 x + \cos^2 x + 2\sin x\cos x}_{(\sin x + \cos x)^2}} = \sqrt{2}\cos 2x$$

$$|\sin x + \cos x| = \sqrt{2}\cos 2x$$

۲

$$\sqrt{2}\left|\sin\left(x + \frac{\pi}{4}\right)\right| = \sqrt{2}\cos 2x$$

$$\xrightarrow{\sin(x + \frac{\pi}{4}) \geq 0} \sin\left(x + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2} - 2x\right)$$

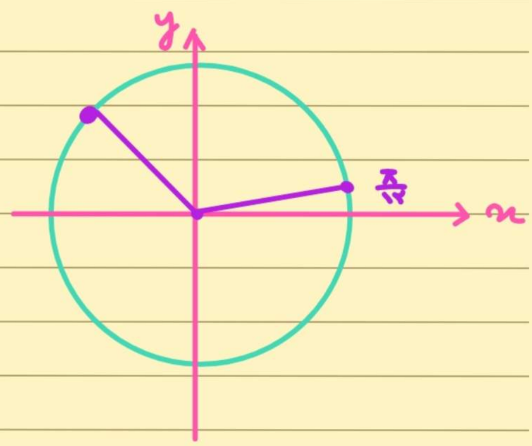
$$x + \frac{\pi}{4} = \frac{\pi}{2} - 2x + 2k\pi$$

$$3x = 2k\pi + \frac{\pi}{4} \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{12} \rightarrow \left\{\frac{\pi}{12}\right\}$$

$$\xrightarrow{\sin(x + \frac{\pi}{4}) < 0} \sin\left(x + \frac{\pi}{4}\right) = \sin\left(2x - \frac{\pi}{2}\right)$$

$$x + \frac{\pi}{4} = 2k\pi + 2x - \frac{\pi}{2}$$

$$x = \frac{\pi}{4} - 2k\pi \rightarrow \left\{\frac{3\pi}{4}\right\}$$



عالی جواب

سارینا عابدینی

حل

الف)

۲

$$\sin^4 x - \cos^4 x = 1$$

$$\sin^2 x - \cos^2 x = 1$$

$$\cos 2x = -1 \qquad 2x = 2k\pi + \pi \qquad x = k\pi + \frac{\pi}{2} \xrightarrow{x \in [0, 2\pi]} x \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$$

ب)

$$\sin^3 x - \cos^3 x = -1$$

بررسی چهار نقطه‌ی اصلی:

$$\sin^3 x - \cos^3 x = -1$$

$x = 0 \rightarrow 0 - (1) = -1$ ✓

$x = \frac{\pi}{2} \rightarrow 1 - (0) = 1$ ✗

$x = \pi \rightarrow 0 - (-1) = 1$ ✗

$x = \frac{3\pi}{2} \rightarrow -1 - (0) = -1$ ✓