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۱) جواب کلی معادلات؟

الف) $\cos 2x = 3\sin x - 1$ $\quad 1 - 2\sin^2 x = 3\sin x - 1$ $\quad 2\sin^2 x + 3\sin x - 2 = 0$

$\Delta = 9 - 4(2)(-2) = 25$

$\sin x = \frac{-3 \pm 5}{4} = \frac{1}{2}$ ✓ $\quad$ یا $\quad -2$ $\quad$ $\sin x = \sin\frac{\pi}{6}$

$x = 2k\pi + \frac{\pi}{6}$

$x = 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6}$

ب) $\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 2\cos^2 x + \cos x - 3 = 0$ $\quad \Delta = 25$

$\cos x = \frac{-1 \pm 5}{4} = \frac{-3}{2}$ یا 1 $\quad x = 2k\pi$

۲)

الف) $\sin^4 - \cos^4 x = \sin 4x$ $\quad (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin 4x$

$-\cos 2x = 2\sin 2x \cos 2x$

$\cos 2x = 0$

$2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\sin 2x = -\frac{1}{2}$ $\quad 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}$

$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$

ب) $2\cos^2 x + 2\sin x \cos x = 1 \quad \sin^2 x + \cos^2 x$ $\quad \cos\left(\frac{\pi}{2} + 2x\right)$

$\cos^2 x - \sin^2 x = -\sin 2x \rightarrow \cos 2x = -\sin 2x$

$2x = 2k\pi + \frac{\pi}{2} + 2x$

$2x = 2k\pi - \frac{\pi}{2} - 2x \rightarrow 4x = 2k\pi - \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8}$

۳)

الف) $\frac{\cos x}{\sin x}\cot x(2\cos x + 1) = \frac{1}{\sin x}$ $\quad \sin x \neq 0$

$\cos x = 0 \quad x = k\pi + \frac{\pi}{2}$

$\cos x = -\frac{1}{2} \quad x = 2k\pi \pm \frac{2\pi}{3}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2$ $\quad \div \sin^2 x \rightarrow 2 - \cot x + \cot^2 x = 2(1 + \cot^2 x)$

$2 - \cot x + \cot^2 x = 2 + 2\cot^2 x \Rightarrow \cot^2 x + \cot x = 0$

$\cot x = 0 \quad k\pi + \frac{\pi}{2} = x$

$\cot x = -1 \quad k\pi - \frac{\pi}{4} = x$

۴) الف) $\cos\left(\frac{\pi}{4}+x\right)\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{4}$ $\quad -x+\frac{\pi}{4}$

$\frac{\pi}{4}+x-x+\frac{\pi}{4}=\frac{\pi}{2}$

$\cos\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}+x\right)=\frac{1}{4}$

$\frac{1}{2}\sin\left(\frac{\pi}{2}+2x\right)=\frac{1}{4}$ $\quad \sin\left(\frac{\pi}{2}+2x\right)=\frac{1}{2} \Rightarrow \cos 2x=\frac{1}{2}$

$2x=2k\pi\pm\frac{\pi}{3} \rightarrow x=k\pi\pm\frac{\pi}{6}$

ب) $\cos\left(2x-\frac{\pi}{9}\right)=-\sin 2x$ $\quad \cos\left(\frac{\pi}{2}+2x\right)$

$2x-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+2x$

$2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \rightarrow 4x=2k\pi-\frac{7\pi}{18} \rightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$

پ) مجموعه جواب معادلات مثلثی: $\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x$

$\frac{\sin 4x+\sin 2x}{\sin 2x}=\sin^2 x+\cos^2 x$ $\quad 1$

$\sin 2x=\sin 4x+\sin 2x$

$\sin 4x=0$

$4x=k\pi \quad x=\frac{k\pi}{4}$

$\sin 2x\neq 0$

$\sin 2x=0$

$2x=k\pi \quad x=\frac{k\pi}{2}$

$x=\frac{k\pi}{2}\pm\frac{\pi}{4}$

ت) دامنه $[-2\pi,\frac{3\pi}{2}]$ دربازه

ت) ثدرمطلق تفاضل جواب های $\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}$

$\frac{\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=1+\cos^2\frac{x}{2}+2\cos\frac{x}{2}$

$2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \rightarrow 2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right)=0$

$\cos\frac{x}{2}=0 \quad \frac{x}{2}=k\pi+\frac{\pi}{2}$

$\cos\frac{x}{2}=-1 \quad \frac{x}{2}=2k\pi+\pi$

$x=4k\pi+2\pi$

$\left|\frac{3\pi}{2}-(-\pi)\right|=\frac{5\pi}{2}$

۷) تعداد جواب‌های $\cos^2 x - \sin^2 x \cos 2x = 1$ در فاصله $[0, 2\pi]$ بیابید.

$\underbrace{\cos^2 x}_{1-\sin^2 x} - \sin^2 x \cos 2x = 1$

$1 - \sin^2 x - \sin^2 x \cos 2x = 1$

$\sin^2 x (1 + \cos 2x) = 0$

$\sin^2 x = 0 \longrightarrow x = k\pi \quad x = 0, \pi, 2\pi$

$1 + \cos 2x = 0 \longrightarrow \cos 2x = -1 \quad 2x = 2k\pi + \pi \quad x = \frac{2k\pi}{2} + \frac{\pi}{2}$

k	0	1	2	3
x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$

۳ جواب

۸) جواب کلی معادله مثلثاتی

$\frac{1 - \tan x}{1 + \tan x} = \tan 2x$

$\tan\left(\frac{\pi}{4} - x\right) = \tan 2x$

$2x = k\pi + \frac{\pi}{4} - x$

$3x = k\pi + \frac{\pi}{4}$

$x = \frac{k\pi}{3} + \frac{\pi}{12}$

۹) معادله مثلثاتی $\sqrt{\underbrace{1 + \sin 2x}_{+}} = \sqrt{2}\cos 2x$ در بازه‌ی $[0, \pi]$ چند جواب دارد؟

$\cos 2x \geq 0$

توان ۲ $\longrightarrow 1 + \sin 2x = 2\cos^2 2x$

$\sin 2x = 2\cos^2 2x - 1$

$\sin 2x = \cos 2x = \sin\left(\frac{\pi}{2} + 2x\right)$

$2x = 2k\pi + \frac{\pi}{2} + 2x$

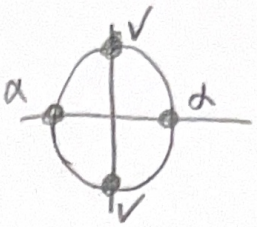
$2x = 2k\pi - \frac{\pi}{2} - 2x$

$4x = 2k\pi - \frac{\pi}{2}$

$x = \frac{k\pi}{2} - \frac{\pi}{8}$

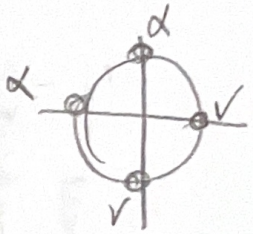
ما) جواب های معادلات مثلثاتی زیر را درباره $[0, 2\pi]$ به دست آورید.

الف) $\sin^4 x - \cos^4 x = 1$



$\frac{\pi}{2}, \frac{3\pi}{2}$

ب) $\sin^3 x - \cos^3 x = -1$



$0, \frac{3\pi}{2}$