Subject
Year　Month　Date　( )

۱۹ آفرین

کیمیا اصیل‌نیا　　تکلیف ۱۶

① $\cos 2x = 3\sin x - 1 \to 1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0$ → $\sin x = -2$ ✗ ، $\sin x = \frac{1}{2}$ ✓

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$\sin x = \sin\frac{\pi}{6}$ → $x = 2k\pi + \frac{\pi}{6}$ ، $x = 2k\pi + \frac{5\pi}{6}$ 　 $k \in \mathbb{Z}$

ب)

$\underbrace{\cos 2x}_{2\cos^2 x - 1} + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$ → $\cos x = -\frac{3}{2}$ ✗ ، $\cos x = 1$ ✓

$\cos x = \cos 2\pi \to x = 2k\pi + 2\pi$ ، $x = 2k\pi - 2\pi$ $\to x = 2k\pi$ 　 $k \in \mathbb{Z}$

② الف)

$\sin^4 x - \cos^4 x = \sin 4x \to -\cos 2x = 2\sin 2x \cos 2x \xrightarrow{\cos 2x \neq 0} \sin 2x = -\frac{1}{2}$

$\sin^2 - \cos^2 = -\cos 2x$

$\cos 2x = 0 = \cos\frac{\pi}{2} \to 2x = 2k\pi \pm \frac{\pi}{2} \to x = k\pi \pm \frac{\pi}{4}$

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$\sin 2x = -\frac{1}{2} = \sin -\frac{\pi}{6} \to 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \pi + \frac{\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

ب)

$2\cos^2 x + 2\sin x\cos x = 1 \to 2\sin x \cos x = 1 - 2\cos^2 x \to \sin 2x = -\cos 2x$

$\to \tan 2x = -1 = \tan -\frac{\pi}{4} \to 2x = k\pi - \frac{\pi}{4} \to x = \frac{k\pi}{2} - \frac{\pi}{8}$

③ الف)

$\cot x(2\cos x + 1) = \frac{1}{\sin x} \to \frac{2\cos^2 x + \cos x}{\sin x} = \frac{1}{\sin x}$

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$\to 2\cos^2 x + \cos x - 1 = 0$ → $\cos x = -1 \to x = 2k\pi + \pi$ ✗ ؛ $\cos x = \frac{1}{2} \to x = 2k\pi \pm \frac{\pi}{3}$

اگر $\cos x = -1$ باشد، $\sin x = 0$ است و عبارت تعریف نشده می‌شود
پس جواب غیر قابل قبول محسوب می‌شود

ب)

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \to 1 + \sin^2 x - \sin x \cos x = 2$

$1 - \sin^2 x = -\sin x \cos x \to \cos^2 x = -\sin x \cos x \xrightarrow{\cos \neq 0} \cos x = -\sin x$

$x = 2k\pi \pm \frac{\pi}{2}$ 　　 $x = k\pi - \frac{\pi}{4}$

۴ الف)

$$\cos(x+\frac{\pi}{4})\underbrace{\cos(x-\frac{\pi}{4})}_{=\cos(\frac{\pi}{4}-x)}=\frac{1}{4} \rightarrow \sin(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})=\frac{1}{4}$$

متمم

$$\frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4} \rightarrow \sin(\frac{\pi}{2}+2x)=\frac{1}{2} \rightarrow \cos 2x=\frac{1}{2}$$

$$2x=2k\pi \pm \frac{\pi}{3} \rightarrow x=k\pi \pm \frac{\pi}{6}$$

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ب)

$$\cos(2x-\frac{\pi}{9})=-\sin 2x \rightarrow \cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}+2x)$$

$$\cancel{2x}-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+\cancel{2x} \;\times$$

$$2x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \rightarrow 4x=2k\pi-\frac{7\pi}{18} \rightarrow x=\frac{k\pi}{2}-\frac{7\pi}{72}$$

۵)

$$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \rightarrow \frac{\sin 4x+\sin 2x}{\sin 2x}=\sin^2+\cos^2=1$$

$$\rightarrow \sin 4x+\sin 2x=\sin 2x \rightarrow \sin 4x=0 \rightarrow 4x=k\pi \rightarrow x=\frac{k\pi}{4}$$

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به ازای برخی مقادیر k مخرج صفر شود

۶)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2} \rightarrow \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

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$$\underbrace{\sin^2\frac{x}{2}}_{1-\cos^2\frac{x}{2}}=1+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \rightarrow 2\cos^2\frac{x}{2}+2\cos\frac{x}{2}=0 \rightarrow 2\cos\frac{x}{2}(\cos\frac{x}{2}+1)=0$$

$$\cos\frac{x}{2}=0 \rightarrow \frac{x}{2}=k\pi+\frac{\pi}{2} \rightarrow x=2k\pi+\pi \quad k\in\mathbb{Z}$$

جواب ها $=-\pi, \pi$

$$\cos\frac{x}{2}=-1 \rightarrow \frac{x}{2}=2k\pi+\pi \rightarrow x=4k\pi+2\pi$$

$$|-\pi-\pi|=2\pi \rightarrow \checkmark$$

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$\cos^2 x - \sin^2 x \cos 3x = 1 \rightarrow \cancel{-\sin^2 x \cos 3x = 1-\cos^2 x} \xrightarrow{\sin^2 x \neq 0} \cos 3x = -1$ ⑦
$\cancel{\sin^2 x}$

$\sin^2 x = 0 \rightarrow \sin x = 0 \rightarrow x = k\pi$

$\cos 3x = -1 \rightarrow 3x = 2k\pi + \pi \rightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$

جواب ها $= 0, \pi, 2\pi, \frac{\pi}{3}, \frac{5\pi}{3}$

۵ جواب

$\frac{1-\tan x}{1+\tan x} = \frac{1}{\tan(x+\frac{\pi}{4})} = \cot(x+\frac{\pi}{4})$

$\frac{1-\tan x}{1+\tan x} = \tan 3x \longrightarrow \cot(x+\frac{\pi}{4}) = \tan 3x = \cot(\frac{\pi}{2} - 3x)$ ⑧

$x+\frac{\pi}{4} = k\pi + \frac{\pi}{2} - 3x \rightarrow 4x = k\pi + \frac{\pi}{4} \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$ ۲

$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{به توان ۲}} 1+\sin 2x = 2\cos^2 2x$ ⑨
$\{ 1-\sin^2 2x$

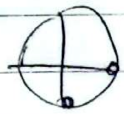
$1+\sin 2x = 2 - 2\sin^2 2x$ ۱/۵

$2\sin^2 2x + \sin 2x - 1 = 0$
$\rightarrow \sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \rightarrow x = k\pi - \frac{\pi}{4}$
$\rightarrow \sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$
$2x = 2k\pi + \pi - \frac{\pi}{6} \rightarrow x = k\pi + \frac{5\pi}{12}$

جواب ها $= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} \rightarrow$ ۳ جواب

به ازای $x=\frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود، این جواب را اضافه نوشته است

(الف) $\sin^4 x - \cos^4 x = 1 \rightarrow -\cos 2x = 1 \rightarrow \cos 2x = -1 \rightarrow 2x = 2k\pi + \pi \rightarrow x = k\pi + \frac{\pi}{2}$ ⑩

$\sin^3 x - \cos^3 x = -1$ تست نقطه‌ها روی دایره مثلثاتی $\rightarrow$  ۲
$x = 2k\pi$
$x = 2k\pi - \frac{\pi}{2}$

سوال ۸

$$\frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \cancel{\sin^2 x + \cos^2 x}^{\,1}$$

$$\frac{\cancel{\sin 2x}(2\cos 2x+1)}{\cancel{\sin 2x}} = 1 \xrightarrow{\sin 2x \neq 0} 2\cos 2x + 1 = 1 \rightarrow \cos 2x = 0$$

$$2x = k\pi + \frac{\pi}{2} \rightarrow \boxed{x = k\pi + \frac{\pi}{4}}$$

سوال ۹

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \xrightarrow{\text{توان ۲}} 1+\sin 2x = 2\cos^2 2x$$

$$1+\sin 2x = 2 - 2\sin^2 2x \rightarrow 2\sin^2 2x + \sin 2x - 1 = 0 \rightarrow \sin 2x = -1 \ , \ \sin 2x = +\frac{1}{2}$$

$$\sin 2x = -1 \rightarrow 2x = 2k\pi - \frac{\pi}{2} \xrightarrow[k=1]{0 \leq x \leq \pi} x = \frac{3\pi}{4} \ ✓$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \ \text{یا} \ 2k\pi + \frac{5\pi}{6} \xrightarrow[k=0]{0 \leq x \leq \pi} x = \frac{\pi}{12} \ ✓ \ \text{یا} \ \frac{5\pi}{12} \ ✗$$

چون عبارت به توان ۲ رسیده است جواب زائد تولید می‌کند

به ازای $x = \frac{5\pi}{12}$ $\cos 2x$ منفی می‌شود و چون جواب رادیکال است [illegible]

* معادله ۲ جواب دارد