نام و نام خانوادگی ...هانیه زهرا شفیعی...... پاسخنامه تشریحی تکلیف شماره ..۱۲... کلاس ..دوازدهم ریاضی..

۱

الف) $\cos 2x = 3\sin x - 1 \to 1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0$

$\to \dfrac{(2\sin x + 4)(2\sin x - 1)}{2} = 0 \to \sin x = -2$ غیرقابل قبول ، $\sin x = \frac{1}{2} \to x \in \{2k\pi + \frac{\pi}{6} , 2k\pi + \frac{5\pi}{6}\}$

ب) $\cos 2x + \cos x - 2 = 0 \to 2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

$\to \cos x = -1 \to x \in 2k\pi + \pi$ ، $\cos x = \frac{3}{2}$ غیرقابل قبول

۲

الف) $\sin^4 x - \cos^4 x = \sin 4x \to (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = \sin 4x \to 1 \times (-\cos 2x) = \sin 4x$

$-\cos 2x = 2\sin 2x \cos 2x \longrightarrow \cos 2x = 0 \to 2x \in 2k\pi \pm \frac{\pi}{2} \to x \in k\pi \pm \frac{\pi}{4}$

$\to \cos 2x \neq 0 \to \sin 2x = \frac{-1}{2} \to 2x \in \{2k\pi + \frac{7\pi}{6} , 2k\pi + \frac{11\pi}{6}\} \to x \in \{k\pi + \frac{7\pi}{12} , k\pi + \frac{11\pi}{12}\}$

ب) $2\cos^2 x + 2\sin x \cos x = 1 \to \sin 2x = 1 - 2\cos^2 x \to \sin 2x = -\cos 2x \to \tan 2x = -1$

$2x = k\pi + \frac{3\pi}{4} \to x = \frac{k\pi}{2} + \frac{3\pi}{8}$

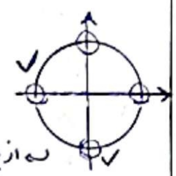
۳

الف) $\cot x (2\cos x + 1) = \dfrac{1}{\sin x} \to \dfrac{\cos x}{\sin x} \times (2\cos x + 1) = \dfrac{1}{\sin x} \to 2\cos^2 x + \cos x - 1 = 0$

$\cos x = -1 \to x \in 2k\pi + \pi$ ، $\cos x = \frac{1}{2} \to x \in 2k\pi \pm \frac{\pi}{3}$

ب) $2\sin^2 x - \sin x \cos x + \cos^2 x = 1 + 1 \to 2\sin^2 x - 1 - \dfrac{\sin 2x}{2} + \dfrac{1 + \cos 2x}{2} = 1$

$\to -\cos 2x - \dfrac{\sin 2x}{2} + \dfrac{1 + \cos 2x}{2} = 1 \to \dfrac{-\cos 2x - \sin 2x + 1}{2} = 1 \to \cos 2x + \sin 2x = -1$



$2x \in \{2k\pi + \pi , 2k\pi - \frac{\pi}{2}\} \to x \in \{k\pi + \frac{\pi}{2} , k\pi - \frac{\pi}{4}\}$ که از بین ۴ نقطه‌ی مرزی، ۲ نقطه‌ی مشخص شده جواب است!

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الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \frac{1}{4} \to \cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4} \xrightarrow{\text{زوایای متمم}} \cos(\frac{\pi}{4} + x) \times \sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\to \dfrac{\sin(\frac{\pi}{2} + 2x)}{2} = \frac{1}{4} \to \cos 2x = \frac{1}{2}$ ، $2x = 2k\pi \pm \frac{\pi}{3} \to x = k\pi \pm \frac{\pi}{6}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x \to \cos(2x - \frac{\pi}{9}) = \sin(-2x) \to \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} - (-2x))$

$\to \cos(2x - \frac{\pi}{9}) = \cos(\frac{\pi}{2} + 2x) \to 2x - \frac{\pi}{9} = \frac{\pi}{2} + 2k\pi + 2x \to -\frac{\pi}{9} = 2k\pi + \frac{\pi}{2}$ غیرقابل قبول

$\to 2x - \frac{\pi}{9} = 2k\pi - 2x - \frac{\pi}{2} \to x = \frac{k\pi}{2} - \frac{7\pi}{72}$

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$\dfrac{\sin 4x + \sin 2x}{\sin 2x} = \sin^2 x + \cos^2 x \to \dfrac{\sin 4x}{\sin 2x} + 1 = 1 \to \dfrac{\sin 4x}{\sin 2x} = 0$

$\sin 4x = 0 \to 4x = k\pi \to x = \frac{k\pi}{4}$ ، $\sin 2x \neq 0 \to 2x \neq k\pi$ ، $x \neq \frac{k\pi}{2} \to$

پس k نباید اعداد زوج باشد چون
به ازای اعداد زوج برای k ، $\frac{k\pi}{4}$
تبدیل به $\frac{k\pi}{2}$ می‌شود و مخرج را صفر می‌کند!

$x \in \{\frac{k\pi}{4}$ ، $k \neq$ اعداد زوج$\}$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}} \to \sin^2\frac{x}{2} = \left(1+\cos\frac{x}{2}\right)^2$$

$$\to \sin^2\frac{x}{2} = 1+\cos^2\frac{x}{2}+2\cos\frac{x}{2} \to \cancel{\sin^2\frac{x}{2}} = \cancel{\sin^2\frac{x}{2}} + \cos^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\cos\frac{x}{2} \to 2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0$$ ۶

$-\pi \leq x \leq \frac{3\pi}{2} \to -\frac{\pi}{2} \leq \frac{x}{2} \leq \frac{3\pi}{4}$ $\cos\frac{x}{2} = -1 \to$ در بازه‌ی داده شده $\cos\frac{x}{2}$ نمی‌تواند ۱- باشد

$\cos\frac{x}{2} = 0 \to \frac{x}{2} = \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\} \to x = \{\pi, -\pi\} \to$ قدر مطلق تفاضل جواب‌ها: $|\pi-(-\pi)| = 2\pi$

$$\cos^2 x - \sin^2 x \cos 2x = 1 \to 1-\cos^2 x = -\sin^2 x \cos 2x \to \sin^2 x = -\sin^2 x \cos 2x$$

$$\to \sin^2 x = 0 \to x \in \{k\pi\}$$

$$\to \sin^2 x \neq 0 \to \cancel{\sin^2 x} = -\cancel{\sin^2 x}\cos 2x \to \cos 2x = -1 \to 2x \in \{2k\pi+\pi\} \to x \in \left\{\frac{2k\pi}{2} + \frac{\pi}{2}\right\}$$ ۷

$$\frac{1-\tan x}{1+\tan x} = \tan 2x \xrightarrow{1=\tan\frac{\pi}{4}} \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\times\tan x} = \tan 2x \to \tan\left(\frac{\pi}{4}-x\right) = \tan 2x$$

$$2x = k\pi + \frac{\pi}{4} - x \to x = \frac{k\pi}{3} + \frac{\pi}{12}$$ ۸

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \to \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x} = \sqrt{2}\cos 2x \to \sqrt{(\sin x+\cos x)^2} = (\cos^2 x-\sin^2 x)\times\sqrt{2}$$

$$\to \sin x+\cos x = \sqrt{2}(\cos x-\sin x)(\cos x+\sin x) \to \sin x+\cos x = 0 \to x =$$

$$\hookrightarrow \sin x+\cos x \neq 0 \to 1 = \sqrt{2}\times(\cos x-\sin x) \to -\sin x+\cos x = \frac{\sqrt{2}}{2}$$ ۹

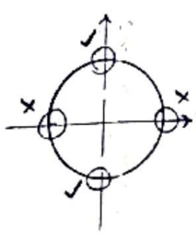
$$\frac{b}{a} = -1 \to \tan\alpha = -1 \to \alpha = \frac{3\pi}{4} \quad \to -\sqrt{2}\sin\left(x+\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} \to \sin\left(x+\frac{3\pi}{4}\right) = \frac{-1}{2}$$

$x+\frac{3\pi}{4} = \frac{7\pi}{6} \to x = \frac{5\pi}{12}$ قابل قبول $\quad x+\frac{3\pi}{4} = \frac{11\pi}{6} \to x = \frac{13\pi}{12}$ غیرقابل قبول در بازه مشخص شده نیست

در قسمت الف و ب در این سوال ما نقاط اصلی را امتحان می‌کنیم!

الف) $\sin^4 x - \cos^4 x = 1$

$x \in \left\{2k\pi \pm \frac{\pi}{2}\right\}$



ب) $\sin^4 x - \cos^4 x = -1$

$x \in \left\{2k\pi, 2k\pi - \frac{\pi}{2}\right\}$ ۱۰