

$$(1) \text{ الف) } 1 - \sqrt{3} \sin^2 u = \sqrt{3} \sin u - 1 \quad \sqrt{3} \sin^2 u + \sqrt{3} \sin u - \sqrt{3} = 0 \rightarrow \sin u = -1 \text{ و } \sin u = \frac{1}{\sqrt{3}}$$

$$u = 2k\pi + \frac{\pi}{3} \quad u = 2k\pi + \pi - \frac{\pi}{3} = 2k\pi + \frac{2\pi}{3}$$

$$\text{ب) } \sqrt{3} \cos^2 u - 1 + \cos u - \sqrt{3} = 0 \quad \cos u = -\frac{\sqrt{3}}{2} \text{ و } \cos u = 1 \quad u = 2k\pi$$

$$(2) \text{ الف) } (\sin^2 u - \cos^2 u)(\sin^2 u + \cos^2 u) = \sin^2 u - \cos^2 u \quad \sin(\frac{\sqrt{3}\pi}{2} + u) = \sin^2 u$$

$$u = 2k\pi + \frac{\sqrt{3}\pi}{2} + u \rightarrow u = k\pi + \frac{\sqrt{3}\pi}{2} \quad u = 2k\pi + \pi - \frac{\sqrt{3}\pi}{2} - u \rightarrow u = \frac{k\pi}{2} - \frac{\pi}{2}$$

$$\text{ب) } \sqrt{3} \cos^2 u + \sqrt{3} \sin u \cos u - 1 = 0 \quad \cos^2 u + \sin^2 u = 0 \quad \sin^2 u = -\cos^2 u$$

$$\sin^2 u = \sin(\frac{\sqrt{3}\pi}{2} + u) \quad u = 2k\pi + \frac{\sqrt{3}\pi}{2} + u \quad u = 2k\pi + \pi - \frac{\sqrt{3}\pi}{2} - u \rightarrow u = \frac{k\pi}{2} - \frac{\pi}{2}$$

$$(3) \text{ الف) } \frac{\cos u}{\sin u} (\sqrt{3} \cos u + 1) = \frac{1}{\sin u} \quad \sin u \neq 0 \rightarrow \cos u (\sqrt{3} \cos u + 1) = 1 \quad \sqrt{3} \cos^2 u + \cos u - 1 = 0$$

$$\cos u = -1 \quad u = 2k\pi + \pi \quad \sin u = 0 \text{ چون}$$

$$\cos u = \frac{1}{\sqrt{3}} \quad u = 2k\pi \pm \frac{\pi}{3}$$

$$\text{ب) } \sqrt{3} (1 - \cos^2 u) - \sin u \cos u + \cos^2 u = \sqrt{3} \quad \sqrt{3} - \sqrt{3} \cos^2 u - \sin u \cos u + \cos^2 u = \sqrt{3}$$

$$-\cos^2 u - \sin u \cos u = 0 \quad -\cos u (\cos u + \sin u) = 0 \quad \cos u = 0 \rightarrow u = k\pi + \frac{\pi}{2}$$

$$\cos u + \sin u = 0 \quad \sqrt{2} \sin(u + \frac{\pi}{4}) = 0 \quad u + \frac{\pi}{4} = k\pi \quad u = k\pi - \frac{\pi}{4}$$

$$(4) \text{ الف) } \cos(u - \frac{\pi}{6} + \frac{\pi}{6}) \cos(u - \frac{\pi}{6}) = \frac{1}{2} \quad -\sin(u - \frac{\pi}{6}) \cos(u - \frac{\pi}{6}) = \frac{1}{2} \quad -\sin(u - \frac{\pi}{6}) = \frac{1}{2}$$

$$\cos u = \frac{1}{2} \quad u = 2k\pi \pm \frac{\pi}{3} \quad u = k\pi \pm \frac{\pi}{6}$$

$$\text{ب) } \cos(u - \frac{\pi}{6}) = \cos(\frac{\pi}{6} + u) \quad u - \frac{\pi}{6} = 2k\pi + \frac{\pi}{6} + u \quad u - \frac{\pi}{6} = 2k\pi - \frac{\pi}{6} - u$$

$$\rightarrow u = \frac{k\pi}{2} - \frac{\pi}{4}$$

$$(5) \frac{\sin^2 u + \sin^2 u}{\sin^2 u} - \sin^2 u = 1 - \sin^2 u \quad \frac{\sin^2 u + \sin^2 u}{\sin^2 u} = 1 \quad \sin^2 u + \sin^2 u = \sin^2 u$$

$$\sin^2 u = 0 \quad \sin^2 u \neq 0 \quad u = k\pi \rightarrow u = \frac{k\pi}{2} \quad u \neq k\pi \quad u \neq \frac{k\pi}{2} \quad u = \frac{k\pi}{2} + \frac{\pi}{2}$$

$$(6) \frac{\sin \frac{u}{2}}{1 + \cos \frac{u}{2}} = \frac{1 + \cos \frac{u}{2}}{\sin \frac{u}{2}} \quad \tan \frac{u}{2} = \cot \frac{u}{2} \quad \tan \frac{u}{2} = \tan(\frac{\pi}{2} - \frac{u}{2}) \quad \frac{u}{2} = k\pi + \frac{\pi}{2} - \frac{u}{2}$$

$$u = 2k\pi + \pi \text{ (I)} \quad \cos \frac{u}{2} \neq -1 \quad \frac{u}{2} \neq 2k\pi + \pi \quad u \neq 4k\pi + 2\pi \text{ (II)} \quad \sin \frac{u}{2} \neq 0 \quad \frac{u}{2} \neq k\pi \quad u \neq 2k\pi \text{ (III)}$$

$$\rightarrow u = 2k\pi + \pi$$

$$(7) \cos^2 u - \sin^2 u \cos^2 u - \cos^2 u - \sin^2 u = 0 \quad -\sin^2 u (\cos^2 u + 1) = 0 \quad \sin u = 0 \quad u = k\pi$$

$$\cos^2 u = -1 \quad u = 2k\pi + \pi \quad u = \frac{2k\pi}{2} + \frac{\pi}{2} \quad u = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$$

جواب

$$\tan(\frac{\pi}{2} - u) = \tan u \quad u = k\pi + \frac{\pi}{2} - u \quad u = \frac{k\pi}{2} + \frac{\pi}{4} \text{ (I)} \quad \tan u \neq -1 \quad u \neq k\pi - \frac{\pi}{4} \text{ (II)}$$

$$\rightarrow u = \frac{k\pi}{2} + \frac{\pi}{4}$$

$$\text{الف) } (\sin^2 u - \cos^2 u)(\sin^2 u + \cos^2 u) = 1 \quad -\cos 2u = 1 \quad \cos 2u = -1 \quad (10)$$

$$2u = 2K\pi + \pi \quad u = K\pi + \frac{\pi}{2} \quad u = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \quad \text{جواب ۲}$$

$$\text{ب) } \underbrace{(\sin u - \cos u)}_{+} \underbrace{(\sin^2 u + \cos^2 u + \sin u \cos u)}_{1 + \frac{1-t^2}{2}} = -1 \quad t\left(\frac{1-t^2}{2}\right) = -1 \quad t^3 - t^2 = -2$$

$$t^3 - t^2 - 2 = (t+1)(t^2 - t - 2) \quad t = -1 \rightarrow \sin u - \cos u = -1 \quad \sqrt{2} \sin\left(u - \frac{\pi}{4}\right) = -1$$

$$\sin\left(u - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \quad u - \frac{\pi}{4} = 2K\pi + \frac{5\pi}{4} \quad u = 2K\pi + \frac{3\pi}{2} \quad u - \frac{\pi}{4} = 2K\pi + \pi - \frac{5\pi}{4}$$

$$u = 2K\pi \quad u = \left\{ 0, \frac{3\pi}{2}, 2\pi \right\} \quad \text{جواب ۳}$$