

(۱) الف) $1 - \sqrt{3} \sin^2 x = \sqrt{3} \sin x - 1 \quad \sqrt{3} \sin^2 x + \sqrt{3} \sin x - \sqrt{3} = 0 \rightarrow \sin x = -1 \text{ و } \sin x = \frac{1}{\sqrt{3}}$
 $x = 2k\pi + \frac{\pi}{3} \quad x = 2k\pi + \pi - \frac{\pi}{3} = 2k\pi + \frac{2\pi}{3}$ ✓
 ب) $\sqrt{3} \cos^2 x - 1 + \cos x - \sqrt{3} = 0 \quad \cos x = -\frac{\sqrt{3}}{2} \text{ و } \cos x = 1 \quad x = 2k\pi$ ✓

(۲) الف) $(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin^2 x - \cos^2 x \quad \sin(\frac{\sqrt{3}\pi}{2} + x) = \sin^2 x$
 $\frac{\sqrt{3}\pi}{2} + x = 2k\pi + \frac{\pi}{2} + x \rightarrow x = 2k\pi + \frac{\sqrt{3}\pi}{2} \quad \frac{\sqrt{3}\pi}{2} + x = 2k\pi + \frac{3\pi}{2} + x \rightarrow x = 2k\pi + \frac{\sqrt{3}\pi}{2}$
 ب) $\sqrt{3} \cos^2 x + \sqrt{3} \sin x \cos x - 1 = 0 \quad \cos^2 x + \sin^2 x = 0 \quad \sin^2 x = -\cos^2 x$
 $\sin^2 x = \sin(\frac{\sqrt{3}\pi}{2} + x)$ ✓
 $x = 2k\pi + \frac{\sqrt{3}\pi}{2} + x \quad x = 2k\pi + \pi - \frac{\sqrt{3}\pi}{2} - x \rightarrow x = \frac{k\pi}{2} - \frac{\pi}{4}$ ✓

(۳) الف) $\frac{\cos x}{\sin x} (\sqrt{3} \cos x + 1) = \frac{1}{\sin x} \quad \sin x \neq 0 \rightarrow \cos x (\sqrt{3} \cos x + 1) = 1 \quad \sqrt{3} \cos^2 x + \cos x - 1 = 0$
 $\cos x = -1 \quad x = 2k\pi + \pi \quad \sin x = 0$ چون ✓
 $\cos x = \frac{1}{\sqrt{3}} \quad x = 2k\pi \pm \frac{\pi}{3}$ ✓
 ب) $\sqrt{3} (1 - \cos^2 x) - \sin x \cos x + \cos^2 x = \sqrt{3} \quad \sqrt{3} - \sqrt{3} \cos^2 x - \sin x \cos x + \cos^2 x = \sqrt{3}$
 $-\cos^2 x - \sin x \cos x = 0 \quad -\cos x (\cos x + \sin x) = 0 \quad \cos x = 0 \rightarrow x = 2k\pi + \frac{\pi}{2}$
 $\cos x + \sin x = 0 \quad \sqrt{2} \sin(x + \frac{\pi}{4}) = 0 \quad x + \frac{\pi}{4} = k\pi \quad x = k\pi - \frac{\pi}{4}$ ✓

(۴) الف) $\cos(x - \frac{\pi}{6} + \frac{\pi}{3}) \cos(x - \frac{\pi}{6}) = \frac{1}{2} \quad -\sin(x - \frac{\pi}{6}) \cos(x - \frac{\pi}{6}) = \frac{1}{2} \quad -\sin(x - \frac{\pi}{6}) = \frac{1}{2}$
 $\cos x = \frac{1}{2} \quad x = 2k\pi \pm \frac{\pi}{3} \quad x = k\pi \pm \frac{\pi}{6}$ ✓
 ب) $\cos(x - \frac{\pi}{6}) = \cos(\frac{\pi}{6} + x) \quad x - \frac{\pi}{6} = 2k\pi + \frac{\pi}{6} + x \quad x - \frac{\pi}{6} = 2k\pi - \frac{\pi}{6} - x$
 $\rightarrow x = \frac{k\pi}{2} - \frac{\pi}{12}$ ✓

(۵) الف) $\frac{\sin^2 x + \sin^2 x}{\sin^2 x} - \sin^2 x = 1 - \sin^2 x \quad \frac{\sin^2 x + \sin^2 x}{\sin^2 x} = 1 \quad \sin^2 x + \sin^2 x = \sin^2 x$
 $\sin^2 x = 0 \quad \sin^2 x \neq 0 \quad x = k\pi \rightarrow x = \frac{k\pi}{2} \quad x \neq k\pi \quad x \neq \frac{k\pi}{2} \quad x = \frac{k\pi}{2} + \frac{\pi}{4}$ ✓

(۶) الف) $\frac{\sin \frac{x}{2}}{1 + \cos \frac{x}{2}} = \frac{1 + \cos \frac{x}{2}}{\sin \frac{x}{2}} \quad \tan \frac{x}{2} = \cot \frac{x}{2} \quad \tan \frac{x}{2} = \tan(\frac{\pi}{2} - \frac{x}{2}) \quad \frac{x}{2} = k\pi + \frac{\pi}{2} - \frac{x}{2}$
 $x = 2k\pi + \pi$ (۱) $\cos \frac{x}{2} \neq -1 \quad \frac{x}{2} \neq 2k\pi + \pi \quad x \neq 4k\pi + 2\pi$ (۲) $\sin \frac{x}{2} \neq 0 \quad \frac{x}{2} \neq k\pi \quad x \neq 2k\pi$ (۳)
 $\rightarrow x = 2k\pi + \pi \quad k \in \mathbb{Z} \rightarrow x = \pi \quad k = -1 \rightarrow x = -\pi$ ✓
 انصاف جواب ها $\leftarrow \pi - (-\pi) = 2\pi$ ✓

(۷) الف) $\cos^2 x - \sin^2 x \cos^2 x - \cos^2 x - \sin^2 x = 0 \quad -\sin^2 x (\cos^2 x + 1) = 0 \quad \sin^2 x = 0 \quad x = k\pi$
 $\cos^2 x = -1 \quad x = 2k\pi + \pi \quad x = \frac{2k\pi}{2} + \frac{\pi}{2}$ ✓
 $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$ ✓
 ب) $\tan(\frac{\pi}{6} - x) = \tan x \quad x = k\pi + \frac{\pi}{6} - x \quad x = \frac{k\pi}{2} + \frac{\pi}{12}$ (۱) $\tan x \neq -1 \quad x \neq k\pi - \frac{\pi}{4}$ (۲)

(۸) $\rightarrow x = \frac{k\pi}{2} + \frac{\pi}{12}$ ✓

$$\text{الف) } (\sin^2 u - \cos^2 u)(\sin^2 u + \cos^2 u) = 1 \quad -\cos 2u = 1 \quad \cos 2u = -1 \quad (10)$$

$$2u = 2k\pi + \pi \quad u = k\pi + \frac{\pi}{2} \quad u = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\} \quad \text{جواب ۲}$$

$$\text{ب) } (\sin u - \cos u)(\sin^2 u + \cos^2 u + \sin u \cos u) = -1 \quad t + \left(\frac{1-t^2}{2} \right) = -1 \quad t^2 - t - 2 = 0$$

$$t^2 - t - 2 = (t+1)(t-2) \quad t = -1 \rightarrow \sin u - \cos u = -1 \quad \sqrt{2} \sin \left(u - \frac{\pi}{4} \right) = -1$$

$$\sin \left(u - \frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \quad u - \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4} \quad u = 2k\pi + \frac{3\pi}{2} \quad u - \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{4}$$

$$u = 2k\pi \quad u = \left\{ 0, \frac{3\pi}{2}, 2\pi \right\} \quad \text{جواب ۳}$$

سوال 9

$$\sqrt{1 + \sin 2n} = \sqrt{2} \cos n \xrightarrow{\text{توان 2}} 1 + \sin 2n = 2 \cos^2 n$$

$$1 + \sin 2n = 2 - 2 \sin^2 n \rightarrow 2 \sin^2 n + \sin 2n - 1 = 0 \rightarrow \sin 2n = -1$$

$$\sin 2n = -1 \rightarrow 2n = 2k\pi - \frac{\pi}{2} \xrightarrow[k \in \mathbb{Z}}{0 \leq n \leq \pi} n = \frac{4k-1}{4}\pi \quad \checkmark$$

$$\hookrightarrow \sin 2n = \pm \frac{1}{2}$$

$$\sin 2n = \frac{1}{2} \rightarrow 2n = 2k\pi + \frac{\pi}{6} \leq 2k\pi + \frac{5\pi}{6} \xrightarrow[k \in \mathbb{Z}}{0 \leq n \leq \pi} n = \frac{2k+1}{12}\pi \leq \frac{5\pi}{12} \quad \checkmark$$

چون عبارت به توان 2 رسیده است جواب زائد تولید میکنند
به ازای $n = \frac{5\pi}{12}$ $\cos 2n$ منفی میشود چنانچه جواب را بیاید است نرد ذات

* معادله 2 جواب دارد