۱

الف) $\cos 2x = 3\sin x - 1 \Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow (\sin x + 2)(2\sin x - 1) = 0$

$\sin x = -2$ غ ق ق ، $-1 \le \sin x \le 1$

$\sin x = \frac{1}{2}$: $\boxed{x = 2k\pi + \frac{\pi}{6}}$ ، $\boxed{x = 2k\pi + \frac{5\pi}{6}}$

ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow (2\cos^2 x - 1) + \cos x - 2 = 2\cos^2 x + \cos x - 3 = 0 \Rightarrow (2\cos x + 3)(\cos x - 1) = 0$

$-1 \le \cos x \le 1$ ، $\cos x = -\frac{3}{2}$ غ ق ق

$\cos x = 1$

$\Rightarrow \boxed{x = 2k\pi}$

۲

الف) $\sin^4 x - \cos^4 x = \sin 4x \Rightarrow \underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = -\cos 2x = \sin 4x \Rightarrow -\sin(\frac{\pi}{2} - 2x) = \sin 4x$

$\Rightarrow \sin(2x - \frac{\pi}{2}) = \sin 4x \Rightarrow$

$2x - \frac{\pi}{2} + 2k\pi = 4x \Rightarrow 2x = 2k\pi - \frac{\pi}{2} \Rightarrow \boxed{x = k\pi - \frac{\pi}{4}}$

$2k\pi + \pi - 2x + \frac{\pi}{2} = 4x \Rightarrow 6x = 2k\pi + \frac{3\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{3} + \frac{\pi}{4}}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \Rightarrow 2\cos^2 x - 1 + 2\sin x\cos x = \cos 2x + \sin 2x = 0 \Rightarrow \sqrt{2}\sin(2x + \frac{\pi}{4}) = 0 \Rightarrow 2x + \frac{\pi}{4} = k\pi \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{\pi}{8}}$

۳

الف) $\cot x(2\cos x + 1) = \frac{1}{\sin x}$ ، $\cot x \neq 0$ $\Rightarrow 2\cos x + 1 = \frac{1}{\underbrace{\cot x}_{\frac{\cos x}{\sin x}} \cdot \sin x} = \frac{1}{\cos x} \Rightarrow 2\cos^2 x + \cos x = 1 \Rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow (\cos x + 1)(2\cos x - 1) = 0$

$\Rightarrow$ $\cos x = -1 \Rightarrow x = 2k\pi + \pi$ غ ق ق *

$\cos x = \frac{1}{2} \Rightarrow \boxed{x = 2k\pi \pm \frac{\pi}{3}}$

* توجه داریم که $\cot x$ باید تعریف شده باشد و همچنین $\sin x$ باید مخالف ۰ باشد.

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \Rightarrow \underbrace{\sin^2 x + \cos^2 x}_{1} + \sin^2 x - \sin x\cos x = 2 \Rightarrow \underbrace{1 - \sin^2 x}_{\cos^2 x} + \sin x\cos x = \cos x(\cos x + \sin x) = 0 \Rightarrow$

$\cos x = 0$

$\sqrt{2}\sin(x + \frac{\pi}{4}) = 0$

$\Rightarrow$ $\boxed{x = k\pi + \frac{\pi}{2}}$

$x + \frac{\pi}{4} = k\pi \Rightarrow \boxed{x = k\pi - \frac{\pi}{4}}$

۴

الف) $\cos(\frac{\pi}{4} + x)\cos(x - \frac{\pi}{4}) = \cos(\frac{\pi}{2} + (x - \frac{\pi}{4}))\cos(x - \frac{\pi}{4}) = -\sin(x - \frac{\pi}{4})\cos(x - \frac{\pi}{4}) = -\frac{1}{2}\sin(2x - \frac{\pi}{2})$

$= +\frac{1}{2}\cos 2x = \frac{1}{4} \Rightarrow \cos 2x = \frac{1}{2} \Rightarrow 2x = 2k\pi \pm \frac{\pi}{3} \Rightarrow \boxed{x = k\pi \pm \frac{\pi}{6}}$

ب) $\cos(2x - \frac{\pi}{9}) = -\sin 2x = \sin(-2x) \Rightarrow \cos(2x - \frac{\pi}{9}) = \cos(\underbrace{\frac{\pi}{2} - (-2x)}_{\frac{\pi}{2} + 2x}) \Rightarrow$

$2x - \frac{\pi}{9} = 2k\pi + 2x + \frac{\pi}{2}$ ✗

$2x - \frac{\pi}{9} = 2k\pi - (\frac{\pi}{2} + 2x)$

$\Rightarrow 4x = 2k\pi - \frac{7\pi}{18} \Rightarrow \boxed{x = \frac{k\pi}{2} - \frac{7\pi}{72}}$

۵

$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x + \sin 2x}{\sin 2x} = \frac{\sin 2x(2\cos 2x + 1)}{\sin 2x} = \underbrace{\cos^2 x + \sin^2 x}_{1} \Rightarrow 2\cos 2x + 1 = 1$ ، $\sin 2x \neq 0$

$\Rightarrow \cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow \boxed{x = \frac{k\pi}{2} + \frac{\pi}{4}}$

۶

$\frac{\sin\frac{x}{2}}{1 + \cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \underbrace{\cot\frac{x}{2}}_{\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}} = \frac{1 + \cos\frac{x}{2}}{\sin\frac{x}{2}} \Rightarrow \tan\frac{x}{4} = \cot\frac{x}{4} = \frac{1}{\tan\frac{x}{4}} \Rightarrow \tan^2\frac{x}{4} = 1 \Rightarrow$

$\tan\frac{x}{4} = 1 \Rightarrow \frac{x}{4} = k\pi + \frac{\pi}{4}$

$\tan\frac{x}{4} = -1 \Rightarrow \frac{x}{4} = k\pi - \frac{\pi}{4}$

$\Rightarrow x = 4k\pi \pm \pi$

$\sin\frac{x}{2} \neq 0$, $\cos\frac{x}{2} \neq -1$

k	۰	۱
x	$-\pi$	3π
	π	5π

$\Rightarrow |\pi - (-\pi)| = \boxed{2\pi}$

۷

$\cos^2 x - \sin^2 x\cos 3x = 1 \Rightarrow \underbrace{1 - \cos^2 x}_{\sin^2 x} + \sin^2 x\cos 3x = 0 \Rightarrow \sin^2 x(1 + \cos 3x) = 0$

$\sin x = 0 \Rightarrow x = k\pi$

$\cos 3x = -1 \Rightarrow 3x = 2k\pi + \pi \Rightarrow x = \frac{2}{3}k\pi + \frac{\pi}{3}$

k	۰	۱	۲	۳
x	0	π	2π	3π
x	$\frac{\pi}{3}$	π	$\frac{5\pi}{3}$	$\frac{7\pi}{3}$

$\boxed{\text{۵ جواب}}$

۸ $\frac{1-\tan x}{1+\tan x} = \tan 3x \Rightarrow \tan(\frac{\pi}{4}-x) = \tan 3x \Rightarrow \frac{\pi}{4} - x + k\pi = 3x \Rightarrow 4x = k\pi + \frac{\pi}{4} \Rightarrow \boxed{x = \frac{k\pi}{4} + \frac{\pi}{16}}$

$\tan x \neq -1 , \sin 3x \neq 0$

۹ $\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \Rightarrow \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = \sqrt{(\sin x + \cos x)^2} = |\sin x + \cos x| = \sqrt{2}(\cos^2 x - \sin^2 x)$

$\Rightarrow |\cos x + \sin x| = \cos x + \sin x \Rightarrow \underbrace{(\cos x + \sin x)}_{\sqrt{2}\sin(x+\frac{\pi}{4})}(1 - \sqrt{2}(\cos x - \sin x)) = 0$

$\sqrt{2}\sin(x+\frac{\pi}{4}) = 0 \Rightarrow x + \frac{\pi}{4} = k\pi \Rightarrow x = k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = \frac{3\pi}{4}}$

$1 = \sqrt{2}\underbrace{(\cos x - \sin x)}_{\sqrt{2}\cos(x+\frac{\pi}{4})} \Rightarrow \cos(x+\frac{\pi}{4}) = \frac{1}{2} \Rightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{3}$

$\Rightarrow x = 2k\pi + \frac{\pi}{12} \Rightarrow \boxed{x = \frac{\pi}{12}} \Longrightarrow$ چون در ناحیه اول است $\sin x + \cos x > 0$ است

$x = 2k\pi - \frac{7\pi}{12}$ در بازه نیست غ ق ق

$|\cos x + \sin x| = -(\cos x + \sin x) \Rightarrow (\cos x + \sin x)(\sqrt{2}\underbrace{(\cos x - \sin x)}_{\sqrt{2}\cos(x+\frac{\pi}{4})} + 1) = 0$

$\cos x + \sin x = 0$ حالت قبلی

$\cos(x+\frac{\pi}{4}) = -\frac{1}{2} \Rightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = 2k\pi + \frac{5\pi}{12} \Rightarrow x = \frac{5\pi}{12}$ غ ق ق در ناحیه اول است پس $\cos x + \sin x > 0$ است

$x = 2k\pi - \frac{11\pi}{12}$ در بازه نیست غ ق ق

$\Rightarrow x = \{\frac{\pi}{12}, \frac{3\pi}{4}\}$ $\boxed{\text{۲ جواب}}$

۱۰ الف) $\sin^4 x - \cos^4 x = 1 \Rightarrow \underbrace{(\sin^2 x + \cos^2 x)}_{1}(\sin^2 x - \cos^2 x) = 1 = -\cos 2x \Rightarrow \cos 2x = -1 \Rightarrow 2x = 2k\pi + \pi \Rightarrow \boxed{x = k\pi + \frac{\pi}{2}}$

k	0	1
x	$\boxed{\frac{\pi}{2}}$	$\boxed{\frac{3\pi}{2}}$

ب) $\sin^3 x - \cos^3 x = (\sin x - \cos x)(\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin x.\cos x) = -1$ $\quad \sin x - \cos x = t \Rightarrow \underbrace{\sin^2 x + \cos^2 x}_{1} - 2\sin x\cos x = t^2$

$\Rightarrow \sin x\cos x = \frac{1-t^2}{2} \Rightarrow t(1 + \frac{1-t^2}{2}) = -1 \Rightarrow t^3 - 3t - 2 = 0 = (t-2)(t+1)^2 \Rightarrow$

$t = 2$ غ ق ق $\quad -\sqrt{2} < \sin x - \cos x < \sqrt{2}$

$t = -1 \Rightarrow \sin x - \cos x = -1$

$\Rightarrow \sqrt{2}\sin(x - \frac{\pi}{4}) = -1 \Rightarrow \sin(x - \frac{\pi}{4}) = -\frac{\sqrt{2}}{2} \Rightarrow$

$x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} \Rightarrow \boxed{x = 2k\pi}$

$x - \frac{\pi}{4} = 2k\pi - \frac{3\pi}{4} \Rightarrow \boxed{x = 2k\pi - \frac{\pi}{2}}$

$\Rightarrow$

k	0	1
x	$\boxed{0}$	$\boxed{2\pi}$
	$-\frac{\pi}{2}$	$\boxed{\frac{3\pi}{2}}$