

۱) الف) $\cos^2 x = 3 \sin^2 x - 1 \Rightarrow 1 - 2 \sin^2 x = 3 \sin^2 x - 1 \Rightarrow 2 \sin^2 x + 3 \sin^2 x - 2 = 0 \Rightarrow (\sin^2 x + 1)(2 \sin^2 x - 1) = 0$

$\sin^2 x = -1$ غلط $\sin^2 x = 1$ $\sin x = \pm 1$

$\sin x = 1 \Rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{2} \\ x = 2k\pi + \frac{5\pi}{2} \end{cases}$

ب) $\cos^2 x + \cos x - 2 = 0 \Rightarrow (\cos^2 x - 1) + \cos x - 2 = 0 \Rightarrow (\cos x + 1)(\cos x - 1) + \cos x - 2 = 0$

$\Rightarrow \cos x = 1$

$\Rightarrow x = 2k\pi$

۲) الف) $\sin^2 x - \cos^2 x = \sin x \Rightarrow (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = -\cos^2 x = \sin x \Rightarrow -\sin(\frac{\pi}{2} - x) = \sin x$

$\Rightarrow \sin(x - \frac{\pi}{2}) = \sin x \Rightarrow \begin{cases} x - \frac{\pi}{2} = x \\ x - \frac{\pi}{2} = \pi - x \end{cases} \Rightarrow \begin{cases} x = 2k\pi - \frac{\pi}{2} \\ x = k\pi + \frac{\pi}{4} \end{cases}$

ب) $2 \cos^2 x + 2 \sin x \cos x = 1 \Rightarrow 2 \cos^2 x - 1 + 2 \sin x \cos x = \cos^2 x + \sin^2 x = 0 \Rightarrow \sqrt{2} \sin(x + \frac{\pi}{4}) = 0 \Rightarrow x + \frac{\pi}{4} = k\pi \Rightarrow x = k\pi - \frac{\pi}{4}$

۳) الف) $\cos x(2 \cos x + 1) = \frac{1}{\sin x} \Rightarrow 2 \cos^2 x + \cos x = \frac{1}{\sin x} \Rightarrow 2 \cos^2 x + \cos x - 1 = 0 \Rightarrow (\cos x + 1)(2 \cos x - 1) = 0$

$\cos x = -1 \Rightarrow x = 2k\pi + \pi$

$\cos x = \frac{1}{2} \Rightarrow x = 2k\pi + \frac{\pi}{3}$

ب) $2 \sin^2 x - \sin x \cos x + \cos^2 x = 2 \Rightarrow \sin^2 x + \cos^2 x + \sin^2 x - \sin x \cos x = 2 \Rightarrow 1 - \sin^2 x + \sin x \cos x = \cos^2 x (\cos x + \sin x) = 0$

$\Rightarrow \begin{cases} \cos x = 0 \\ \sqrt{2} \sin(x + \frac{\pi}{4}) = 0 \end{cases}$

$\Rightarrow \begin{cases} x = k\pi + \frac{\pi}{2} \\ x + \frac{\pi}{4} = k\pi \Rightarrow x = k\pi - \frac{\pi}{4} \end{cases}$

۴) الف) $\cos(\frac{\pi}{6} + x) \cos(x - \frac{\pi}{6}) = \cos(\frac{\pi}{6} + (x - \frac{\pi}{6})) \cos(x - \frac{\pi}{6}) = -\sin(x - \frac{\pi}{6}) \cos(x - \frac{\pi}{6}) = -\frac{1}{2} \sin(2x - \frac{\pi}{3})$

$= \pm \frac{1}{2} \cos 2x = \frac{1}{2} \Rightarrow \cos 2x = 1 \Rightarrow 2x = 2k\pi \Rightarrow x = k\pi$

ب) $\cos(2x - \frac{\pi}{4}) = -\sin 2x = \sin(-2x) \Rightarrow \cos(2x - \frac{\pi}{4}) = \cos(\frac{\pi}{4} - 2x) \Rightarrow \begin{cases} 2x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \\ 2x - \frac{\pi}{4} = 2k\pi - (\frac{\pi}{4} + 2x) \end{cases}$

$\Rightarrow \begin{cases} 2x = 2k\pi + \frac{\pi}{2} \\ 2x = 2k\pi \end{cases} \Rightarrow \begin{cases} x = k\pi + \frac{\pi}{4} \\ x = k\pi \end{cases}$

۵) $\frac{\sin^2 x + \sin x}{\sin x} - \sin^2 x = \cos^2 x \Rightarrow \frac{\sin^2 x \cos^2 x + \sin x}{\sin x} = \frac{\sin^2 x (2 \cos^2 x + 1)}{\sin x} = \cos^2 x + \sin^2 x \Rightarrow 2 \cos^2 x + 1 = 1$

$\Rightarrow \cos^2 x = 0 \Rightarrow x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$

۶) $\frac{\sin \frac{\pi}{4}}{1 + \cos \frac{\pi}{4}} = \frac{1}{\sin \frac{\pi}{4}} + \cot \frac{\pi}{4} = \frac{1 + \cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} \Rightarrow \tanh \frac{\pi}{4} = \cot \frac{\pi}{4} = \frac{1}{\tanh \frac{\pi}{4}} \Rightarrow \tanh^2 \frac{\pi}{4} = 1 \Rightarrow \begin{cases} \tanh \frac{\pi}{4} = 1 \Rightarrow \frac{\pi}{4} = k\pi + \frac{\pi}{4} \\ \tanh \frac{\pi}{4} = -1 \Rightarrow \frac{\pi}{4} = k\pi - \frac{\pi}{4} \end{cases}$

$\Rightarrow x = 2k\pi \pm \frac{\pi}{2}$

$\sin \frac{\pi}{4} \neq 0, \cos \frac{\pi}{4} \neq -1$

۷) $\cos^2 x - \sin^2 x \cos^2 x = 1 \Rightarrow 1 - \cos^2 x + \sin^2 x \cos^2 x = 0 \Rightarrow \sin^2 x (1 + \cos^2 x) = 0$

$\sin^2 x = 0 \Rightarrow x = k\pi$

$\cos^2 x = -1 \Rightarrow x = 2k\pi + \pi \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{2}$

k	0	1	2	3
x	0	π	2π	3π
x	π/2	3π/2	5π/2	7π/2

$$\frac{1 - \tan^2 n}{1 + \tan^2 n} = \tan^2 n \Rightarrow \tan^2 \left(\frac{\pi}{4} - n \right) = \tan^2 n \Rightarrow \frac{\pi}{4} - n + k\pi = n \Rightarrow n = k\pi + \frac{\pi}{4} \Rightarrow \boxed{n = \frac{k\pi}{4} + \frac{\pi}{4}}$$

$\tan n \neq -1, \sin^2 n \neq 0$

$$\sqrt{1 + \sin^2 n} = \sqrt{2} \cos n \Rightarrow \sqrt{\sin^2 n + \cos^2 n + 2 \sin n \cos n} = \sqrt{(\sin n + \cos n)^2} = |\sin n + \cos n| = \sqrt{2} (\cos^2 n - \sin^2 n)$$

$$\Rightarrow \begin{cases} |\cos n + \sin n| = \cos n + \sin n \Rightarrow (\cos n + \sin n) (1 - \sqrt{2} (\cos n - \sin n)) = 0 \\ \Rightarrow \begin{cases} \cos n + \sin n = 0 \Rightarrow \sin(n + \frac{\pi}{4}) = 0 \Rightarrow n + \frac{\pi}{4} = k\pi \Rightarrow n = k\pi - \frac{\pi}{4} \Rightarrow \boxed{n = \frac{k\pi}{4} - \frac{\pi}{4}} \\ 1 - \sqrt{2} (\cos n - \sin n) = 0 \Rightarrow \cos(n + \frac{\pi}{4}) = \frac{1}{\sqrt{2}} \Rightarrow n + \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \end{cases} \end{cases}$$

دو طرفه است $\sin n + \cos n$ است

$$\Rightarrow \begin{cases} n = 2k\pi + \frac{\pi}{4} \Rightarrow \boxed{n = \frac{\pi}{4}} \\ n = 2k\pi - \frac{\pi}{4} \Rightarrow \boxed{n = -\frac{\pi}{4}} \end{cases}$$

دو طرفه است $\cos n + \sin n$ است

$$|\cos n + \sin n| = -(\cos n + \sin n) \Rightarrow (\cos n + \sin n) (\sqrt{2} (\cos n - \sin n) + 1) = 0$$

$$\Rightarrow \begin{cases} \cos n + \sin n = 0 \Rightarrow \sin(n + \frac{\pi}{4}) = 0 \Rightarrow n + \frac{\pi}{4} = k\pi \Rightarrow n = k\pi - \frac{\pi}{4} \Rightarrow \boxed{n = \frac{k\pi}{4} - \frac{\pi}{4}} \\ \sqrt{2} (\cos(n + \frac{\pi}{4}) - \sin(n + \frac{\pi}{4})) = -1 \Rightarrow \cos(n + \frac{\pi}{4}) = -\frac{1}{\sqrt{2}} \Rightarrow n + \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4} \end{cases}$$

دو طرفه است $\cos n + \sin n$ است

$$\Rightarrow \begin{cases} n = 2k\pi + \frac{3\pi}{4} \Rightarrow n = \frac{3\pi}{4} \\ n = 2k\pi - \frac{3\pi}{4} \Rightarrow n = -\frac{3\pi}{4} \end{cases}$$

دو طرفه است $\cos n + \sin n$ است

$$\Rightarrow n = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\} \quad \boxed{\text{جواب ۲}}$$

الف) $\sin^2 n - \cos^2 n = 1 \Rightarrow (\sin^2 n + \cos^2 n) (\sin^2 n - \cos^2 n) = 1 = -\cos 2n \Rightarrow \cos 2n = -1 \Rightarrow 2n = 2k\pi + \pi \Rightarrow \boxed{n = k\pi + \frac{\pi}{2}}$

k	0	1
n	$\frac{\pi}{2}$	$\frac{3\pi}{2}$

ب) $\sin^2 n - \cos^2 n = (\sin n - \cos n) (\sin n + \cos n) = -1$ $\sin n - \cos n = t \Rightarrow \sin^2 n + \cos^2 n - 2 \sin n \cos n = t^2$

$$\Rightarrow \sin n \cos n = \frac{1 - t^2}{2} \Rightarrow t \left(1 + \frac{1 - t^2}{2} \right) = -1 \Rightarrow t^3 - 2t - 2 = 0 = (t - 2)(t + 1)^2 \Rightarrow \begin{cases} t = 2 \Rightarrow \sqrt{2} (\sin n - \cos n) = \sqrt{2} \\ t = -1 \Rightarrow \sin n - \cos n = -1 \end{cases}$$

$$\Rightarrow \sqrt{2} \sin \left(n - \frac{\pi}{4} \right) = -1 \Rightarrow \sin \left(n - \frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \Rightarrow \begin{cases} n - \frac{\pi}{4} = 2k\pi - \frac{\pi}{4} \Rightarrow \boxed{n = 2k\pi} \\ n - \frac{\pi}{4} = 2k\pi - \frac{3\pi}{4} \Rightarrow \boxed{n = 2k\pi - \frac{\pi}{2}} \end{cases}$$

k	0	1
n	0	$\frac{\pi}{2}$
	$-\frac{\pi}{2}$	$\frac{3\pi}{2}$